Age-of-mortality functions as a new indicator of longevity extension in high-income countries

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1 Introduction

Over the last decades in many high-income countries the age pattern of mortality shifted to older ages, a phenomenon referred to as the 'aging of mortality decline' (Horiuchi and Wilmoth, 1995; Wilmoth, 1997). During this period, death rates at older ages dropped faster than at younger ages and mortality above age 80 started to decline considerably for the first time (Kannisto et al., 1994; Thatcher et al., 1998; Wilmoth, 1997). Therefore, the extension in the length of human life has been primarily fueled by survival improvement at older ages (Meslé and Vallin, 2006; Vallin and Meslé, 2001; Wilmoth et al., 2000) and future longevity gains will depend exclusively on the mortality decline among the elderly. Hence, it has become increasingly important that trends and differentials in longevity over the last decades be monitored through the lens of indicators that are specific to aging.

If longevity extension is defined simply as the ability of individuals to survive to increasingly higher ages, such as reflected by age shifts of the old-age mortality curve, then what constitutes 'old' ages? There are at least three approaches to the operational definition of 'old' ages. They can be represented 1. by a fixed age range specific to aging, for example ages above 65, 2. by the location of the old-age death heap in the distribution of deaths, or 3. by ages at which death rates correspond to particular values of high adult mortality. It is important to note that under the first approach, the 'old' age range remains fixed. In contrast, under the second and third approach, the 'old' age range is dynamically defined, that is, an improvement in old-age survival moves mortality levels to higher ages.

Based on the approaches mentioned above, three potential measures can be used for monitoring the longevity gains made over the last decades: 1. the life expectancy conditional to survival to age 65, e_{65}, 2. the late modal age at death, M, and 3. the ages corresponding to particularly high levels of old-age mortality. As discussed in the methodology section below, the ages at which specific death rates are observed are derived from the inverse function of age-specific mortality, labeled as age-of-mortality (AoM) functions. The advantage of these three indicators is that they can be measured on the age-scale (i.e. length of human life), thus making them comparable over time. Conditional life expectancy at age 65 and, more recently, the modal age at death, M, have been widely used to monitor the longevity extension in low mortality countries since the early 1970s. In contrast, the AoM functions have not been yet considered as possible indicators of longevity. However, these functions have been used in the past for measuring mortality compression (Ediev, 2013, 2014) and as a useful tool for mortality projections (Ishii, 2015).

Therefore, this paper is the first to investigate the use of AoM functions as possible measures of longevity extension. We do so by analyzing trends and differentials in AoM functions at levels of high adult mortality in four low-mortality countries, namely Canada, France, Japan, and the US over the 1970-2011 period. For the purpose of our study, we used death rates which are currently representative of old-age mortality, distinguishing three levels: $0.1e^{-0.5}$ ("low"), 0.1 ("moderate"), and $0.1e^{0.5}$ ("high"). Our research will provide insight on how the AoM functions at the selected levels of old-age of mortality evolved over time and at what pace. Moreover, the comparison with trends in e_{65} and in M will allow to determine if these two lifespan indicators need to be accompanied by other longevity measures in order to have a complete picture of the changes that occurred in the length of human life over the past decades.

2 Data and Methods

2.1 Data

Observed death counts and population exposure by single year of age, calendar year, and sex for Canada, France, Japan and the United States covering the 1971-2010 period were taken from the Human Mortality database (HMD, 2018).

2.2 Methods

For a given calendar year and sex,

1. the life expectancy at age 65, e_{65}, is given by:

$$e_{65} = \int_{65}^{\omega} exp\Big[-\int_{65}^{a} \mu(u)du\Big]da, \tag{1}$$

   where $\mu(x)$ represents the age-specific force of mortality at age x and ω is the open-aged interval.

2. The modal age at death, M, representing the age at which the highest proportion of deaths occur, is obtained by maximising the density function $f(x)$. That is,

$$M = \max_x \ f(x) = \max_x \mu(x)S(x) = \max_x \mu(x)exp\Big[-\int_{0}^{x} \mu(u)du\Big]. \tag{2}$$

3. The age-of-specific mortality (AoM) function, denoted by $\nu(y)$, is defined as:

$$\nu(y) = \lambda^{-1}(y) \tag{3}$$

   where $\lambda(x) = log(\mu(x))$. Under the assumption that mortality at older ages increases monotonically over age x, i.e. $\lambda(x) = \lambda_x$, then the log-force of mortality, has an inverse, denoted by $\nu(y)$. A graphical representation of the log-force of mortality and its corresponding inverse can be found in Figure 1.

The force of mortality $\mu(x)$ appearing in the calculation of the three chosen measures of longevity extension, equations (1)-(3), was estimated using a flexible nonparametric smoothing method based on P-splines. The P-spline approach has been proven highly effective for smoothing mortality rates and hence for obtaining smoothed age patterns of mortality (Camarda, 2008, 2012; Currie et al., 2004). The P-spline approach for Poisson death counts has been widely used in mortality studies in the past years (Diaconu et al., 2016; Ebeling et al., 2018; Horiuchi et al., 2013; Ouellette and Bourbeau, 2011; Ouellette et al., 2012a,b; Remund et al., 2018). When it is applied to old-age mortality data, it usually ensures that the estimated force of mortality increases monotonically with age.

3 Results

Figure 2 illustrates changes in the ages at which the selected values of AoM functions are observed, i.e. $\nu(0.1e^{-0.5})$, $\nu(0.1)$, and $\nu(0.1e^{0.5})$, in life expectancy at age 65, e_{65}, and in modal age at death, M, for Canada, France, Japan, and US over the 1970-2011 period. It reveals an upward trend in all five measures of 'old' ages as well as important differences in the ages attained by each measure. In general, the following ranking (from lowest to highest) can be observed: $\nu(0.1e^{-0.5})$, e_{65}, $\nu(0.1)$, M, and $\nu(0.1e^{0.5})$.

Comparative analysis of trends and differentials in AoM functions at selected levels of old-age mortality, in e_{65}, and in M revealed significant differences in the pace at which these ages increased. In fact, $\nu(0.1e^{-0.5})$ increased at a faster pace than $\nu(0.1)$ which in turn exhibited a more rapid pace of change than $\nu(0.1e^{0.5})$. In Canada, for example, the AoM function at a level of $0.1e{-}0.5$ increased by about 5.5 years throughout the 1970-2011 period, as $\nu(y)$ went from 79.5 to 85 years. However, the pace of change in the ages at which the selected moderate- and high-level of old-age mortality were observed was more modest, about 4.5 and three years. However, the ages at which a death rate of 0.1 and $0.1e^{0.5}$ were 5 and 10 years higher than the ones attained by $\nu(0.1e^{-0.5})$ in 1970. Recent studies showed that the tendency for a smaller increase in $\nu(y)$ at higher ages is an indication of old-age mortality compression (Ediev, 2013, 2014). In the US, the pace of increase in the ages corresponding to the three selected levels of old-age mortality was similar to the Canadian one.

In France and in Japan, $\nu(0.1e^{-0.5})$ attained 78 and 85 years in 1970 and 85.6 and 89 years in 2011 resulting in an increase of 7 and 10 years throughout the 40-year period. As was the case in Canada, the shift to older ages of the AoM functions at the 0.1 and $0.1e^{0.5}$ levels was more modest, respectively of 6 and 5.5 years in France and of 8 years in Japan. A more in-depth comparison of the pace of increase in the three $\nu(y)$s revealed that, the increment in $\nu(0.1e^{-0.5})$ was almost 70% larger than the one in $\nu(0.1e^{0.5})$, in Canada and in the US, and reached approximately 30% and 20%, in France and in Japan.

How do trends in $\nu(0.1)$ compare to those in M and e_{65}? Throughout the study period, the ages corresponding to a log mortality level of 0.1 were bounded by the values of e_{65}, downward, and by those of M upwards. The shift in $\nu(0.1)$ to higher ages was smaller than the one in M. The divergence in the trends of these two measures of 'old' ages more pronounced since the mid-1970s in Japan, the mid-1980s in Canada and in France, and the early-2000s in the US. Throughout the 40-year period, the increase in the age at which the highest proportion of deaths occurred ranged between 5.5 years (the US) and 10 years (Japan). The gain in M over the 1970-2011 period represented about 13% of the gain in $\nu(0.1)$, in France and in Japan, and amounted to about 35% and 40% in the US and Canada. In contrast, the number of years gained in e_{65} over the 40-year period was more modest, varying between 3.5 years (the US) and 8 years (Japan). The modest increase in e_{65} is consistent with the mathematical finding that if the mortality schedule above age x shifts to higher ages by about u years, then the increase in life expectancy at age x is less than u years (Horiuchi et al., 2013). The differences observed between the trends in $\nu(0.1)$ and the two conventional lifespan indicators was somewhat surprising given that the 0.1 mortality level is generally observed around M and $65 + e_{65}$. A closer analysis of our results revealed that the pace of increase as well as the shape of the trend in $\nu(0.1e^{-0.5})$ are quite similar to those in M while those in $\nu(0.1)$ resemble those in $\nu(0.1)$.

4 Conclusion

Our preliminary results showed that the three measures of longevity chosen followed an upward trend since the 1970s in each of the countries studied. Moreover, it revealed that trends in AoM functions are systematically different from those in e_{65} and in M. These findings seem to suggest that the three measures of longevity studied show different aspects of the gains made in the length of life during the ‘aging of mortality decline’ period. In fact, the increase observed in e_{65}, in M, and in the AoM function at selected old-age mortality levels indicate that over time survival improved in the fixed old-age range of 65+, that the old-age death heap moved to higher ages, and that high levels of adult mortality shifted to older ages. Our findings suggests that the use of AoM functions, in addition to the conventional lifespan indicators, is likely to provide a deeper insight on the longevity gains made over the last decades following the remarkable decline in old-age mortality. Therefore, the age-of-mortality function should be used more extensively in future longevity research.

5 Future work

This is an ongoing project and we intend to conduct the same analysis for females in order to determine if the results are similar to those obtained for males. Moreover, we plan on estimating the same results for other low-mortality countries in oder to see if the results can be generalized to other populations. We will also investigate why changes in the shape of the trend as well as in the pace of M are comparable to those observed at ages of relatively small levels of old-age mortality while those in e_{65} resemble more those observed at ages of relatively high levels of old-age mortality. While all the indicators studied followed an upward trend, we also wonder if the pace of increase accelerated, decelerated or remained the same throughout the 40-year period. We will investigate this either by smoothing the trend, by fitting some nonlinear function (i.e. parabolic), or by multiple-slope regression models.

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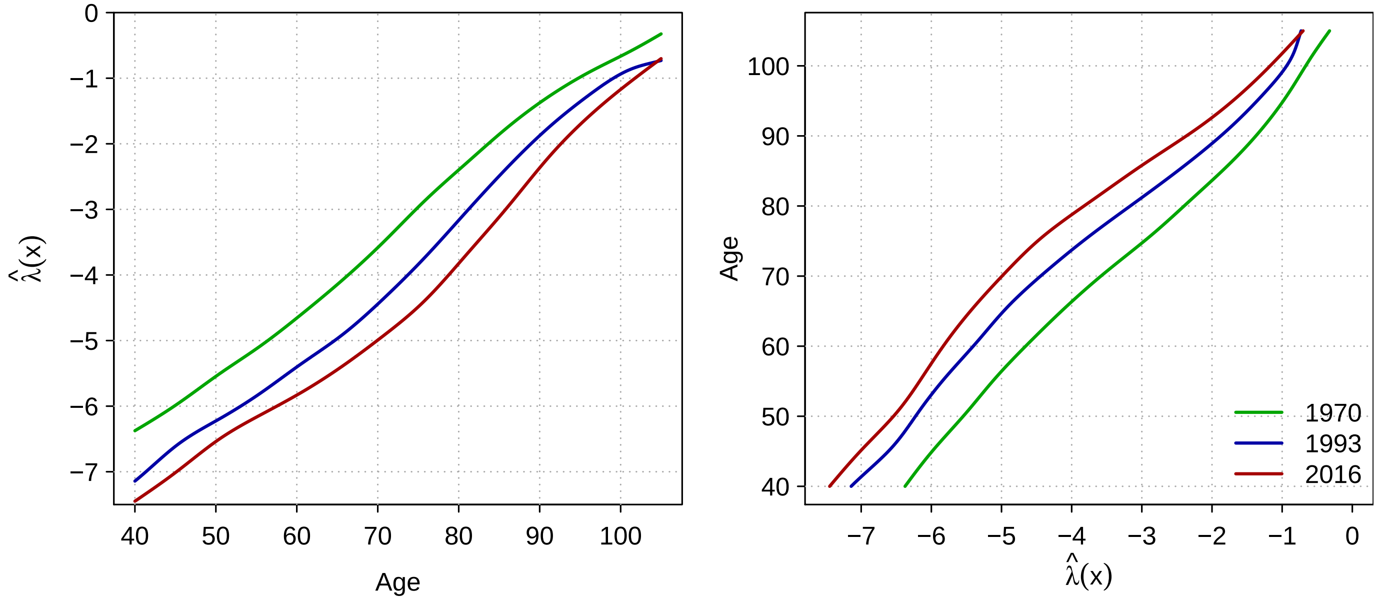
Appendix

Figure 1: Log-force of mortality, $\hat{\lambda}(x)$, and the inverse of the log-force of mortality resulting from nonparametric P-splines smoothing, Japan, selected calendar years between 1971 and 2016

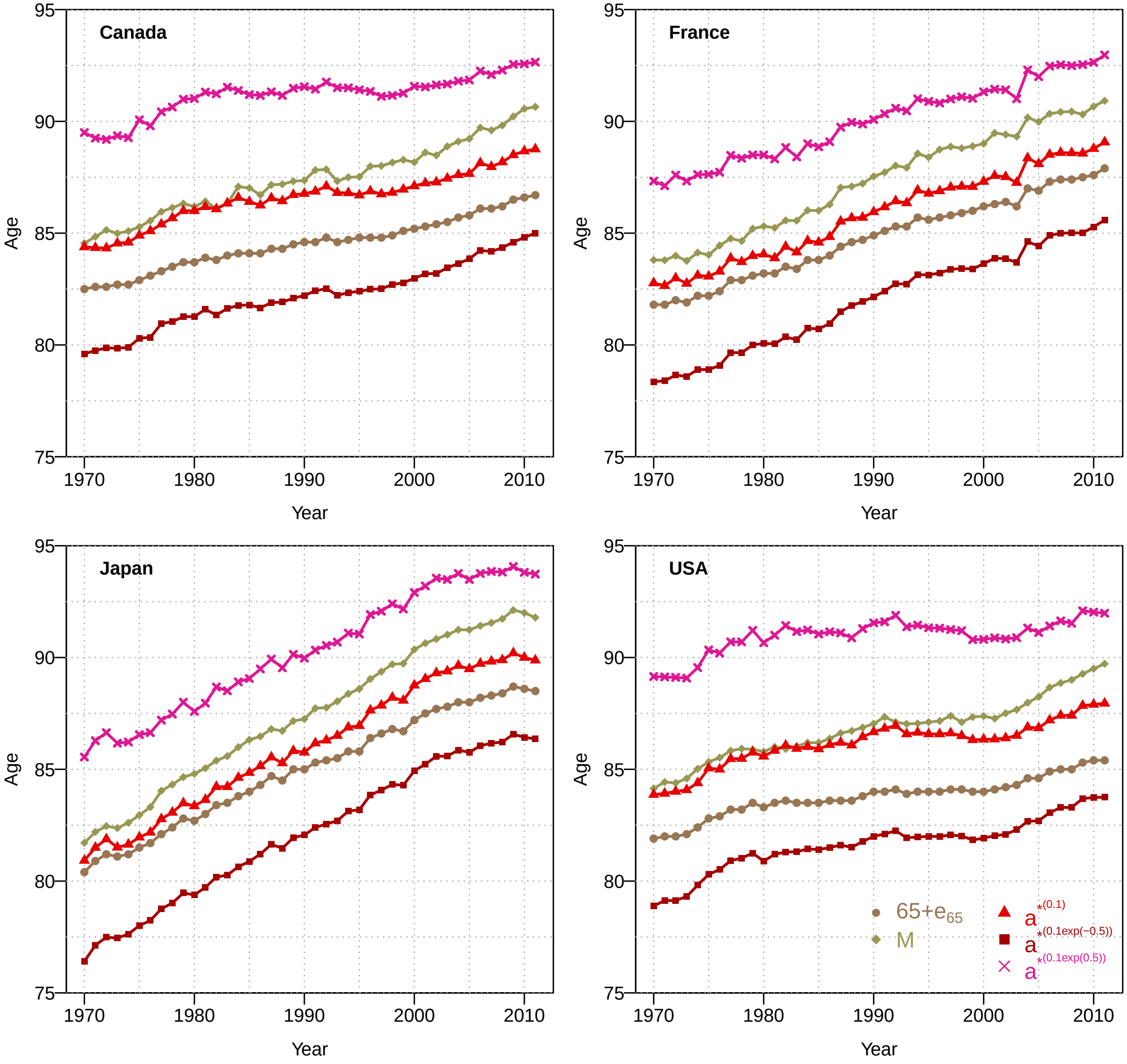
MALES



Note: A comparable pair of graphs for Japanese females has previously been shown by Ishii (2015)
Source: Authors' calculations

Figure 2: Total life expectancy at age 65 ($65 + e_{65}$), modal age at death (M) and AoM functions at selected old-age mortality levels for Canada, France, Japan, and the US, 1970-2011

MALES



Source: Authors' calculations