Rising Tides Lift Which Boats?
Connecting Absolute and Relative Mobility Across Generations

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September 2018

Abstract

In assessing the extent to which individuals escape childhood disadvantages (or maintain childhood advantages), researchers often study relative mobility across generations (individuals' movements up or down the income rankings from their parents' position). Yet many people experience absolute income gains across generations without any relative mobility. This paper explores the connection between relative and absolute mobility. I show how population-level mobility patterns connect to individual-level mobility experiences and expand upon prior work on absolute mobility to show how this mobility depends on not only economic growth and income inequality but also relative mobility. I propose nonparametric and parametric approaches to characterizing the joint distribution of absolute and relative mobility as experienced by individuals. Using National Longitudinal Survey of Youth data, I describe how mobility experiences differ across people from (dis)advantaged backgrounds, paying special attention to racial variation in the probability of experiencing upward absolute mobility without upward rank mobility.

Extended abstract (draft in progress).
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Extended Abstract

In assessing the extent to which individuals escape childhood disadvantages (or maintain childhood advantages), researchers often study relative mobility across generations, that is, individuals' movements up or down the income rankings from their parents' position. Joseph Schumpeter (1955) famously compared relative mobility to a hotel in which some rooms are luxurious and others are shabby; the rooms are always occupied, but the occupants change over time. When studying relative mobility, we fixed the positions available and examine the comparative chances of individuals from different backgrounds of moving into these positions. In contrast, absolute mobility captures any change in income across generations; it is not limited to comparative shifts. Returning to the hotel metaphor, we can imagine that everyone in the hotel remains in their rooms but every room is upgraded to the same degree, such that the least desirable rooms remain the least desirable compared to other currently available rooms, but they are more desirable than they were before the renovation. In this situation, we would observe absolute mobility but no relative mobility. Absolute mobility is not sufficient for relative mobility; nor is it necessary.[1] Individuals can replicate their parents' incomes exactly yet fall down the income rankings, if their cohort peers have increased their incomes across generations.

Although, historically, relative mobility has received more scholarly attention than absolute mobility from researchers interested in inequality of opportunity (due to its comparative nature; Breen and Jonsson 2005), absolute mobility has long been viewed as important for understanding both the extent to which economic growth is widely shared (Sawhill and Morton 2007) and the ways in which individuals assess their economic wellbeing (in light of the expectations that they formed in the parental home; Easterlin 1987). Clearly, then, relative and absolute mobility provide different, yet related, insights into how economic (dis)advantages evolve across generations.[2] What complementarities are there between relative and absolute mobility, and what trade-offs? How often do (groups of) people experience absolute mobility without relative mobility, or vice versa? Are initially advantaged people more likely to move up in both ranks and raw dollars, while initially disadvantaged people are more likely to move up only in dollars but remain stuck in in low rankings?

In this paper, I propose a new approach toward measuring the transformation of childhood (dis)advantages into adult (dis)advantages, focused on characterizing the

[1] Likewise, relative mobility is neither sufficient nor necessary for absolute mobility.

[2] Similarly, the processes driving relative and absolute mobility are distinct but, generally, nonindependent. For example, while absolute mobility is typically tied to large-scale economic shifts like industrialization and relative mobility is typically tied to individual characteristics like educational attainment, the expansion of secondary and then post-secondary education should have affected opportunities for both absolute and relative mobility.

joint distribution of absolute and relative mobility (and how this joint distribution varies across initially advantaged versus disadvantaged subgroups). Along the way, I make three contributions to the literature. First, I expand upon recent work that used simulation methods to describe how absolute mobility, measured at the population level as the share of adults whose income exceeds their parents' income, varies in response to economic growth and income inequality (Chetty et al. 2017). I show how this population-level absolute mobility also varies in response to relative mobility, also measured at the population level, using a straight-forward analytic approach. Second, I describe how these population-level patterns reflect different individual-level experiences of absolute mobility among people from low- and high-income backgrounds. Third, I propose nonparametric and parametric approaches toward characterizing the joint distribution of absolute and relative mobility, as experienced on an individual level. Using data from the National Longitudinal Survey of Youth, I will estimate the joint distribution and how it varies across the parental income distribution as well as by racial group, nativity, childhood family structure, and education. I anticipate that people from relatively advantaged backgrounds and social groups will be particularly likely to experience upward mobility in both absolute dollars and rank positions, while people from relatively disadvantaged backgrounds will be more likely to experience absolute upward mobility without gains in relative rankings.

Analytic Approach

In what follows I provide an outline of the analytic approaches I employ to, first, expand on prior research on absolute upward mobility at the population level (which examined how this mobility varies with average economic growth and changes in income inequality) by also showing how this mobility varies with relative mobility at the population level. I then outline how I relate this population-level analysis to, second, individual-level analysis of absolute mobility and its relation to population relative mobility and, third, individual-level analysis of the joint distribution of absolute and relative mobility.

Let $Y = \log(\text{Income})$ and $R = \text{rank}(\text{Income})$. Note that because the log transformation is rank preserving, $R = \text{rank}(Y)$ as well. Standard analyses of intergenerational income mobility focus on the slope coefficient from a regression of log adult income on log parental income,

$$Y_i^a = \alpha + \beta Y_i^p + \epsilon_i \tag{1}$$

where income is often adjusted for age, measurement error, and family size. β captures intergenerational persistence; its complement $(1 - \beta)$ captures mobility. Traditionally, researchers transformed incomes with logs, allowing β to be interpreted as an intergenerational income elasticity (Solon 1999). An elasticity captures both relative persistence (similarity in parents' and children's relative income positions) and differences in income inequality across generations.

$$\beta = \rho \frac{\sigma_a}{\sigma_p} \tag{2}$$

ρ is the intergenerational correlation of log incomes; it ranges between zero and one in practice, although in theory it ranges from -1 to 1 as is true of any correlation. I take ρ as my population-level measure of relative mobility.[3] When ρ is closer to zero, relative mobility is higher; when ρ is closer to one, relative mobility is lower (and, equivalently, the persistence of relative income positions across generations is higher).

At the population level, we can measure upward absolute mobility as the share of adults whose income exceeds their parents' income. (We can also extend this to incorporate different thresholds of mobility, measuring, for example, the share of adults whose incomes are at least 10% higher than their parents' income.) The share of adults whose income exceeds their parents' income is equal to the share of adults whose logged income exceeds their parents' logged income, because the log transformation is rank preserving. At a population level, this share can be thought of as a probability,

$$\underbrace{A_0}_{\substack{\text{Absolute Mobility} \\ \text{Share Gaining} \\ >0\% \text{ Relative} \\ \text{to Parents}}} = \frac{\sum_{i=1}^{N} 1(Y_i^a - Y_i^p > 0)}{N} = P(Y_i^a - Y_i^p > 0) \tag{3}$$

where 1() is the indicator function taking on the value 1 if the condition is true and 0 otherwise. Assuming that logged adult and parental income follow a bivariate normal distribution with mean $\mu = (\mu_a, \mu_p)$ and covariance matrix Σ with diagonal values of $(\sigma_a^2,\, \sigma_p^2)$ and off-diagonal values of $\rho\sigma_a\sigma_p$, such that ρ captures the intergenerational correlation (as discussed above), then it is simple to derive the fact that

$$A_0 = \Phi\left(\frac{\mu_a - \mu_p}{\sqrt{\sigma_a^2 + \sigma_p^2 - 2\rho\sigma_a\sigma_p}}\right) \tag{4}$$

where $\Phi()$ is the standard normal cumulative distribution function. This formulation can be easily extended to capture different types threshold of upward absolute mobility. Let Δ be the threshold, such that we capture the (expected) share of people whose

[3] Note that the intergenerational correlation in income ranks, ρ_r, is closely related to the intergenerational correlation in log incomes, ρ. When log income is distributed bivariate normal and ρ is small, $\rho \approx 1.05\rho_r$ (Trivedi and Zimmer 2007).

incomes exceed their parents' income by Δ percent or more with A_Δ.

$$\underbrace{A_\Delta}_{\substack{\text{Absolute Mobility}\\ \text{Share Gaining}\\ >\Delta\%\text{ Relative}\\ \text{to Parents}}} = \Phi\left(\frac{\mu_a - \mu_p}{\sqrt{\sigma_a^2 + \sigma_p^2 - 2\rho\sigma_a\sigma_p}} - [e^\Delta - 1]\right) \tag{5}$$

I will use these formulas to draw conclusions about how upward absolute mobility varies as a function of relative mobility. These formulas indicate that when there is more relative mobility (i.e., when ρ is closer to zero), there is less upward absolute mobility (i.e., A_Δ is smaller). I will examine different values of the key parameters to assess the magnitude of this relationship and how it compares with other relationships (e.g., with average growth, $\mu_a - \mu_p$, and with inequality in the parental and adult generations, σ_p^2 and σ_a^2).

I will then, in my second contribution, move to understanding how these population-level dynamics reflect different absolute mobility experiences at the individual level among people from low- and high-income backgrounds. I define individual-level experienced absolute mobility as the difference between parental and adult income.

$$\underbrace{Y_i^a - Y_i^p}_{\substack{\text{Experienced}\\ \text{Absolute Mobility}}} = \underbrace{(\hat{Y}_i^a - Y_i^p)}_{\substack{\text{Expected}\\ \text{Absolute Mobility}}} + \underbrace{(Y_i^a - \hat{Y}_i^a)}_{\substack{\text{Unexpected}\\ \text{Absolute Mobility}}} \tag{6}$$

I decompose this experienced absolute mobility into a piece that is expected based on parental income and a piece that is unexpected. The expected piece is based on the adult income value predicted from the model in eq. 1. Eq. 2 shows how this expected adult income is related to relative mobility as well as inequality in the parental and adult generations. Eq. 7 (below) show how it also depends on average income growth across generations.

$$\alpha = \mu_a - \rho\frac{\sigma_a}{\sigma_p}\mu_p \tag{7}$$

Using these facts, simple manipulation reveals that we can understand the expected portion of experienced absolute mobility as a function of average income growth, a mean reversion that differs across people based on how far from the mean they begin (with larger moves the further from the mean they begin), and a halted mean reversion (with lower relative mobility leading to more upward absolute mobility among people starting from high-income backgrounds).

$$\underbrace{\hat{Y}_i^a - Y_i^p}_{\substack{\text{Expected}\\ \text{Absolute Mobility}}} = \underbrace{(\mu_a - \mu_p)}_{\substack{\text{Shared}\\ \text{Growth}}} - \underbrace{(Y_i^p - \mu_p)}_{\substack{\text{Mean}\\ \text{Reversion}}} + \underbrace{\rho \frac{\sigma_a}{\sigma_p}(Y_i^p - \mu_p)}_{\substack{\text{Halted}\\ \text{Reversion}}} \tag{8}$$

Similarly, we can understand unexpected absolute mobility as a function of relative mobility and inequality, with the implications of different parameter values differing across the parental income distribution.

$$\underbrace{Y_i^a - \hat{Y}_i^a}_{\substack{\text{Unexpected}\\ \text{Absolute Mobility}}} = \underbrace{(Y_i^a - \mu_a)}_{\substack{\text{Mean}\\ \text{Deviation}}} - \underbrace{\rho \frac{\sigma_a}{\sigma_p}(Y_i^p - \mu_p)}_{\substack{\text{Halted}\\ \text{Reversion}}} \tag{9}$$

Table 1 summarizes how individual-level experienced absolute mobility varies with relative mobility at the population level, differently for people from low- and high-income backgrounds. I will connect these patterns to the population-level pattern documented in eq. 3–5 above.

Finally, I propose two approaches toward characterizing the joint distribution of absolute and relative mobility at the individual level. Analogously to how I define experienced absolute mobility, I define experienced relative mobility as the difference between individuals' ranks in the adult income distribution and their parents' income rank in their generation's income distribution. A simple, nonparametric measure of how absolute and relative mobility covary can be read off of a graph plotting experienced absolute mobility against experienced relative mobility, as in Figure 1. At the individual level, of course, we expect experienced relative mobility and experienced absolute mobility to be positively correlated.[4] Yet the share of people falling into each quadrant of Figure 1 will provide important information about who benefits in which ways from economic growth and rising inequality. People who are upwardly mobile both absolutely and relatively will be in the upper-right quadrant, while people who are downwardly mobile in both dimensions will be in the lower-left. The off-diagonal quadrants reveal

[4]

$$\text{Cov}\,[\underbrace{Y^a - Y^p}_{\substack{\text{Experienced}\\ \text{Absolute}\\ \text{Mobility}}}, \underbrace{R^a - R^p}_{\substack{\text{Experienced}\\ \text{Relative}\\ \text{Mobility}}}] = \underbrace{(\text{Cov}[Y^a, R^a] + \text{Cov}[Y^p, R^p])}_{\substack{\text{Covariances}\\ \text{Within Generations}}} - \underbrace{(\text{Cov}[Y^a, R^p] + \text{Cov}[Y^p, R^a])}_{\substack{\text{Covariances}\\ \text{Across Generations}}} \tag{10}$$

The sum of the covariances within generations should exceed the sum of the covariances across generations, leading the overall covariance to be positive, per the above equation. Experienced relative mobility is also positively correlated with both expected absolute mobility and unexpected absolute mobility. However, the correlation between experienced relative mobility and expected absolute mobility increases with population relative mobility (i.e., that correlation is more positive when ρ is closer to zero). In contrast, the correlation between experienced relative mobility and unexpected absolute mobility decreases with population relative mobility (i.e., that correlation is less positive when ρ is closer to zero).

who floats with the rising tide (experiencing upward absolute mobility but downward relative mobility) and who emerges from a receding tide (experiencing upward relative mobility but downward absolute mobility). I will compare the shares of people in these quadrants from different advantaged and disadvantaged backgrounds. Importantly, this measure can be easily compared across groups that are typically difficult to compare in terms of relative mobility because they regress to different group-specific means.

To obtain more precise insight into how relative and absolute mobility vary together (evidencing complementarities or trade-offs), as well as how this variation differs across people from different backgrounds and/or demographic groups, I introduce a covariance regression model that jointly parameterizes the mean and covariance structure of the bivariate outcome, experienced absolute mobility, and experienced relative mobility; $\boldsymbol{Y_i} = (Y_i^a - Y_i^p, R_i^a - R_i^p)$. Drawing on the model introduced by Bloome and Schrage (2018), I separate the covariance structure into a model on variances and a model on the correlation between outcomes.

$$\boldsymbol{Y_i} \sim \text{Bivariate Normal}(\boldsymbol{\mu_i}, \boldsymbol{\Sigma_i}) \tag{11}$$

$$\boldsymbol{\mu_i} = \boldsymbol{X_i}\boldsymbol{\beta} \tag{12}$$

$$\boldsymbol{\Sigma}_i = \begin{bmatrix} \sigma_{i1}^2 & \rho_{i12}\sigma_{i1}\sigma_{i2} \\ \rho_{i21}\sigma_{i2}\sigma_{i1} & \sigma_{i2}^2 \end{bmatrix} \tag{13}$$

$$\log \sigma_i^2 = \boldsymbol{X_i}\boldsymbol{\gamma} \tag{14}$$

$$\rho_i = 2 \times \text{logit}^{-1}(\boldsymbol{X_i}\delta) - 1 \tag{15}$$

$$\tag{16}$$

$\boldsymbol{Y_i}$ is a 1×2 matrix of outcomes, and i goes from $1 \dots n$, where n is the number of observations. $\boldsymbol{X_i}$ is a $1 \times q$ matrix of predictors, where q is the number of predictors (including an intercept). $\boldsymbol{\beta}$ is a $q \times 2$ matrix of mean regression coefficients. $\boldsymbol{\gamma}$ is a $q \times 2$ matrix of variance regression coefficients. δ is a q-vector of correlation regression coefficients, modeling the correlation between the two outcomes. $\boldsymbol{\Sigma_i}$ here is a covariance matrix of σ_i and ρ_i variances and correlations. The covariance matrix is specific to each observation; values of $\boldsymbol{Y_i}$ for a particular observation i depend on μ_i and $\boldsymbol{\Sigma_i}$ (where μ_i is a 2-vector with one mean per outcome for each observation, and $\boldsymbol{\Sigma_i}$ is composed of the variances $\{\sigma_{i1}^2, \sigma_{i2}^2\}$ and the correlations ρ_i. A Bayesian estimation approach allows for straight-forward inference on all quantities of interest related to how relative and absolute mobility differ across $\boldsymbol{X}$ and covary with one another (also differently across

$\boldsymbol{X}$).

Data and Expected Findings

I will examine data from the National Longitudinal Survey of Youth, 1979 cohort, to study empirical patterns of absolute and relative mobility in family income, as well as how these mobility patterns vary across people from low- and high-income backgrounds and by racial group, nativity, childhood family structure, and education. For information on the data and key data cleaning choices, see Bloome (2017) and Bloome et al. (2018). I anticipate that people from relatively advantaged backgrounds and social groups will be particularly likely to experience upward mobility in both absolute dollars and rank positions, while people from relatively disadvantaged backgrounds will be more likely to experience absolute upward mobility without gains in relative rankings. I will pay special attention to how mobility among black men and women may have been more consequential in terms of absolute dollars than in terms of relative income positions (and the power that such positions entail).

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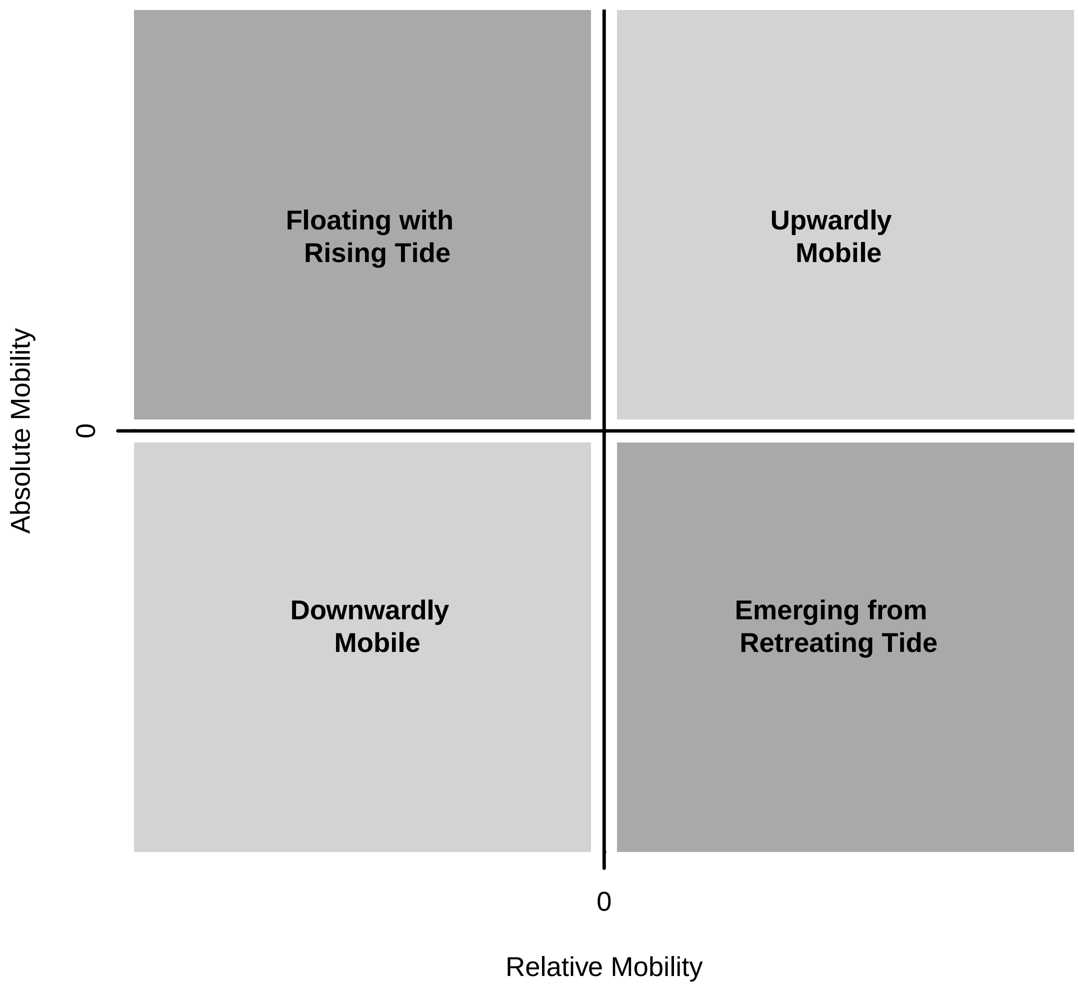

Figure (1) Joint distribution of absolute mobility and relative mobility, measured at individual level.

Table (1)

	Individual Absolute Mobility	
Parental Inc.	Expected	Unexpected
Above mean	Decreases with relative mobility	Increases with relative mobility
Below mean	Increases with relative mobility	Decreases with relative mobility

Note: "Increase" indicates positive movement, while "decrease" indicates negative movement. Relative mobility is higher when ρ is closer to zero.