Tilting vs. Stretching: the relative importance of fertility and mortality on population aging over the long-term

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Abstract

I present comparative statics relationships for the stretching of the age structure due to longer life comparable to those that exist for the tilting of the age structure from changing growth rates. I use these relationships to understand why increasing longevity appears to be such an important force in influencing the fiscal aspects of aging.

1 Introduction

Around the world, populations will age dramatically over the next decades. Commentators universally point to change in mortality and fertility. Fertility had long been thought to be the main driver of population aging But in recent decades the importance of mortality has also been emphasized (Grazielli, Preston ...). In this paper, I present some new methods for understanding the demographic importance of fertility and mortality. I take as an illustrative example the fiscal implications of population aging in stable populations.

Comparative static calculations showing the effect of changing growth rates have long been a standard part of the demographic toolkit. Beginning with Lotka's (1939) *Théorie analytique des associations biologiques,* the main applications of comparative statics were fully developed in Keyfitz's (1977) *Applied Mathematical Demography.* In economic demography, Lee's earlier work on transfers (1994) as well as the recent (2014) publication in *Science* both make extensive use of comparative statics.

In the past, demographers have been motivated by the dramatic effects of the demographic transition on population growth rates. They have used stable population theory to show the effect of changes in fertility[1] on age structure. Changes in growth rates effectively "tilt" the age pyramid, with fast growing populations having many young and few old, and fast shrinking populations having many old relative to young.

In modern populations, low fertility is still a source of change in age-structure. However, now increases in longevity are also an important driver of population aging. Longer life "stretches" the age pyramid, despite having little or no effect on steady-state population growth rates. In this paper, I show how comparative static calculations can also be performed for stylized changes in longevity. The methods for mortality are not as simple, nor perhaps as aesthetically pleasing, as the results for growth rates. However, it is possible to provide an approximate result that is simple and has intuitive appeal.

It is also possible to show using these methods that the effect of increasing longevity interacts with fertility levels, such that at lower levels of fertility, changes in longevity will have even larger effects.

This paper is organized as follows. First, I begin by summarizing the results of our analysis, presenting the formula for the comparative statics of growth and longevity side-by-side and providing an illustrative application. Second, I then present a derivation of the new formula for the comparative statics of longevity. I then conclude with a brief discussion of the interpretation of the comparative statics results.

A main finding of this analysis is that expected longevity increases are likely to have much larger effects than plausible variation in fertility.

In particular, I provide some caveats to the interpretation that longevity change is quantitatively more important than fertility-induced changes in the growth rates.

2 Comparative statics for growth and longevity

In a stable population, the number of people aged x is proportional to $\ell(x)e^{-rx}$, where $\ell(x)$ is the fraction of newborns surviving to age x, r is

[1] and infant mortality via its influence on net fertility

the stable growth rate, and the quantity e^{-rx} accounts for the exponential change in birth cohort size over time. Larger growth rates will increase the fraction that are young; smaller growth rates will increase the fraction that are old. In the following we will consider the case where $\ell(0) = 1.0$.

Now consider an age-schedule $w(x)$ telling us the behavior or characteristics of people aged x. In a stable population the total of this characteristic for the population with $\ell(0) = 1$ can be written as

$$W(r) = \int \ell(x) e^{-rx} w(x)\, dx. \tag{1}$$

We can express W as a function of the growth rate r and a parameter θ that influences survival.

$$W(r, \theta) = \int \ell(x)^{\theta} e^{-rx} w(x)\, dx, \tag{2}$$

where values of $\theta > 1$ will reduce life expectancy and values of $\theta < 1$ will increase life expectancy. In what follows, for brevity, when we consider θ, we will let $r = 0$, and when r is of interest, we will let $\theta = 1$.

Comparative statics calculations for the "effect" of changing r or θ on W are then obtained by taking the derivative of W with respect to the variable of interest. It is of particular interest to consider the effect on the logarithm of W, since this tell us the proportional effect, which is then independent of the scale or units of W. We then obtain the classic result (e.g., Keyfitz) that the effect of changing r will depend on the mean age. Holding the number aged zero constant,

$$\frac{d \log W}{dr} = -\frac{\int x w(x) \ell(x) e^{-rx}\, dx}{\int w(x) \ell(x) e^{-rx}\, dx} = -A_w. \tag{3}$$

For changes in longevity induced by *proportional changes in age-specific death rates*, as we will derive in the next section, I find

$$\frac{d \log W}{de_0} = \frac{1}{W} \frac{\int \log \ell(x) \ell(x) w(x)\, dx}{\int \log \ell(x) \ell(x)\, dx} \approx \frac{w(e_0)}{W}. \tag{4}$$

Here we shown the result directly with respect to life expectancy e_0. As we will see, this approximation is particularly accurate when the change in life expectancy is due to a proportional change in Gompertz hazards.

The change in a particular quantity W could represent the amount of taxes paid, the amount of health care needed, the demand for golf-courses, the number of subscribers to a magazine, the number of widows, or any other quantity that is meaningful to express as a function of age-specific rates. In many cases it is of interest to consider the ratio of two such quantities. This ratio could be that of produces to consumers, payers to receivers, students to teachers, retirees to workers, or any other quantity of interest for which changes in the numbers at different ages are of interest.

Consider the specific example of the fiscal support ratio, defined as

$$FSR = \frac{\int n(x)t(x)\,dx}{\int n(x)b(x)\,dx}, \tag{5}$$

where $n(x)$ is the number of persons aged x, $t(x)$ is the average taxes paid by age and $b(x)$ is the average benefits received (or government "cost") by age. For ratios such as that for fiscal support, the relevant comparative statics relationships for stable populations are

$$\frac{d \log FSR(r)}{dr} = A_b - A_t, \tag{6}$$

for the growth rate; and for the effect of longevity,

$$\frac{d \log FSR(e_0)}{de_0} \approx \frac{t(e_0)}{T} - \frac{b(e_0)}{B}, \tag{7}$$

where $T = \int \ell(x)t(x)\,dx$ and $B = \int \ell(x)b(x)\,dx$.

How are we to use these relationships? Typically, the mean age of benefit is about 10 years older than the mean age of tax payment. This tells us that a 1 percent change in the growth rate will – in the long-run – produce about a 10 percent change in the ratio of taxes to benefit. Changing the number of surviving children from 2.0 to 1.5 – covering the difference in projected long-term fertility between, say, Germany and France, will lower the growth rate by about 1 percent. The long-term consequence of this fertility change will be to create a fiscal shortfall of about 10 percent.

For longevity, in Europe life expectancy is increasing by something like 2 years per decade. Over the course of a half century, this will be about a 10 year increase in longevity. Typically in a modern economy, the fraction of life time benefits received at the age equal to life expectancy is about 3 percent, whereas the fraction of taxes paid at this age is less than 1 percent. This means that a 10 year increase in longevity would cause about a 20 percent fiscal shortfall, roughly twice the magnitude of the fertility change. This larger impact of longevity increase will grow as longevity increases, whereas the effect of "low" fertility would be a one-time change, unless birth rates were to sink further. More on the comparison between the relative importance of fertility and mortality will be found in the discussion at the end of this note.

Before proceeding to the derivation, it is fair to ask how these comparative static calculations compare to "real" population projections. Taking the case of Germany and France, the fiscal support ratios for these two countries – using a common schedule – is about 1.0 today and is projected to fall to about 0.8 in Sweden (France?) and about 0.7 in Germany (based on current age-schedules of taxes and benefits and Eurostat population projections). This is consistent with the comparative statics results. We would not expect the relationship to be exact both because our comparative statics results are only approximate for larger than infinitesimal changes, and because actual population age structures subject to variation in vital rates will not have stable age structures.

3 Derivations

The derivation of the classic result on growth rates is straightforward, obtained as given in equation (6) by taking the derivative with respect to r. For longevity, the result in equation (7) is less transparent. Both the relationship between θ and life expectancy e_0 and the nature of the approximation needs to be made explicit.

Begin by noting that the relationship between fiscal support and life expectancy can be seen as operating via changes in age-specific survival and the parameter θ. Specifically,

$$\frac{dW}{de_0} = \frac{\frac{dW}{d\theta}}{\frac{de_0}{d\theta}}. \tag{8}$$

The effect of changing θ on W is given by[2]

$$\frac{dW}{d\theta} = \int \ell(x)^\theta \log \ell(x) w(x)\, dx \tag{9}$$

[2] In what follows I show the results for stationary populations with age structure proportional to $\ell(x)$. They could be extended to cover stable populations with non-zero growth rates at the cost of a bit more notation.

The effect of θ on life expectancy is given by the well-known "entropy" relationship in mortality research, namely,

$$\frac{de_0}{d\theta} = \int \ell(x) \log \ell(x) \, dx, \tag{10}$$

where here and afterwards, without loss of generality, we omit θ from the right hand side, equivalent to letting $\theta = 1$ for an appropriately defined baseline survival schedule.

We can define the distribution over age of person years gained or lost from a change in θ as

$$h(x) = \frac{\ell(x) \log \ell(x)}{\int \ell(x) \log \ell(x) \, dx}. \tag{11}$$

We can then write

$$\frac{dW}{de_0} = \int h(x) w(x) \, dx. \tag{12}$$

The above result is exact and tells us that the effect of changing life expectancy will be the sum of the effects of changing survival at each age, weighted by profile of interest. We can also approximate this weighted sum using the usual Taylor series approximations of moments of a function of a random variable, since $h(x)$ is a probability density and $w(x)$ is a function of x. To first order, the approximation would be

$$\frac{d \log W}{de_0} \approx \frac{w(\bar{x})}{W}, \tag{13}$$

where $\bar{x}$ is the mean age of h. This approximation is exact when w is linear. To evaluate the effect of curvature in the profile, one can use the second order approximation,

$$\frac{d \log W}{de_0} \approx \frac{1}{W} \left[w(\bar{x}) + \frac{w''(\bar{x}) \sigma_h^2}{2} \right],$$

where σ_h^2 is the variance of h and w'' is the curvature of the profile. We can see that the extra term here is in proportion to the degree of non-linearity of the weighting function and h's spread over ages.

How are we to think about the distribution h? The denominator of h tells us the total change in life expectancy, or expected person-years lived, caused by a change in θ, while the numerator tells us the effect at each age. So, one way to think of the distribution h is as the relative contribution of change in survival at each age to total changes in life expectancy. In general the distribution h looks rather like the distribution of ages of death, with its mean being slightly lower than life expectancy because of the higher relative importance of survival at younger ages. However, if we neglect mortality at younger ages and consider the case of hazards being Gompertzian, the distribution h is nearly identical to the distribution of $\ell(x)\mu(x)$ for the age of death, and so $\bar{x} = e_0$.[3]

The Gompertz case is a good model for contemporary mortality change appropriate when there is little young mortality and age-specific decline is fairly uniform over all ages. For the Gompertz case, equation (13) simplifies to

$$\frac{dW}{de_0} \approx w(e_0). \tag{14}$$

Here we have arrived at an expression that is easily interpretable. The effect of changing life expectancy will be proportional to the value of w at the age equal to the average length of life of the population.

This lengthy derivation makes explicit the assumptions that produce our result for the effect of increases in longevity on population-weighted totals. Equation (12) shows the exact effect. However, the approximate effect shown in equation (14) can also be arrived at much more simply. Imagine that all deaths were concentrated at age e_0, making the value of $\ell(x) = 1$ for $x < e_0$. In this case, using the fundamental theorem of calculus, we obtain the exact relation,

$$\frac{dW}{de_0} = \frac{d}{de_0}\int_0^{e_0} 1 \cdot w(x)\, dx = w(e_0). \tag{15}$$

This approach of concentrating a distribution upon its mean is useful to understand what is driving the "stretching" effect of increasing longevity. It can also be used to understand the "tilting" effect of changing growth rates. If we concentrate, say, all taxes at the mean age of taxation and all benefits at the mean age of benefit, then we obtain the exact result given in equation (6).

[3]This is true when the α term of the Gompertz hazard model $\mu(x) = \alpha \exp(\beta x)$ is small relative to β, as is the case in modern populations. To show that $\ell(x)\mu(x) \approx \ell(x) \log \ell(x) / \int \ell(x) \log \ell(x)\, dx$, it suffices to show that $\mu(x) \approx \log \ell(x) / \int \ell(x) \log \ell(x)\, dx$. Substituting the Gompertz hazard gives $\log \ell(x) / \int \ell(x) \log \ell(x)\, dx = (\mu(x) - \alpha)/(1 - \alpha/\beta e_0)$. As $\alpha \to 0$, the quantity $\log \ell(x) / \int \ell(x) \log \ell(x)\, dx$ approaches $\mu(x)$.

4 Applications

4.1 Mean ages of stable populations

The mean age of the stable population is

$$A(r, \theta) = \int a\ell(a)^\theta e^{-ra}\, da.$$

The standard result from stable population theory is

$$\frac{\partial A}{\partial r} = -\sigma_a^2,$$

where the right hand side is the variance of age in the stable population.

For changes in life expectancy, applying our approach gives

$$\frac{\partial A}{\partial e0} = \frac{(\bar{x} - A)}{e_0},$$

where $\bar{x}$ is the mean age of the $h(x)$ distribution. If everyone died at the same age e_0, then the mean age in the population would be $e_0/2$ and $\frac{dA}{de_0} = \frac{1}{2}$. In low mortality populations $\bar{x}$ is close to $e0$. So, we can see that our result will also be close to 1/2.

To illustrate, Swedish females had a life expectancy of 72.4 years according to the period lifet able of 1950. The mean age of a stationary population with this life table would be 37.8 years. In this case, we would then obtain $\frac{\partial A}{\partial e0} = (72.4 - 37.8)/72.4 = 0.48$. This is quite close to 1/2.

4.2 Application to NTA schedules

We now apply our formal results to the observed age schedules of goverment revenue (taxes) and expenditures (benefits) found in a number of countries by Lee et al. (2014). Our goal is two-fold. First we are interested in comparing the relative effects of changing fertility (the growth rate) and changing mortality (life expectancy). We will be surprised by how much mortality matters, compared to fertility for fiscal issues. Second, we are interested evaluating the accuracy of our comparative statics approximations.

Figure 4.2 shows the results of simulated changes in mortality and fertility on the Fiscal Support Ratio of the United States. The upper right panel shows the $t(x)$ tax and $b(x)$ benefit schedules by age as calculated by the

National Transfer Accounts project. The vertical grey line is the value of e_0 that is used in the approximation. Gompertz mortality with constant log-slope is used for the simulation, with changes in life expectancy obtained by changing the intercept of the hazard curve. The resulting survival curves are shown in the upper right panel.

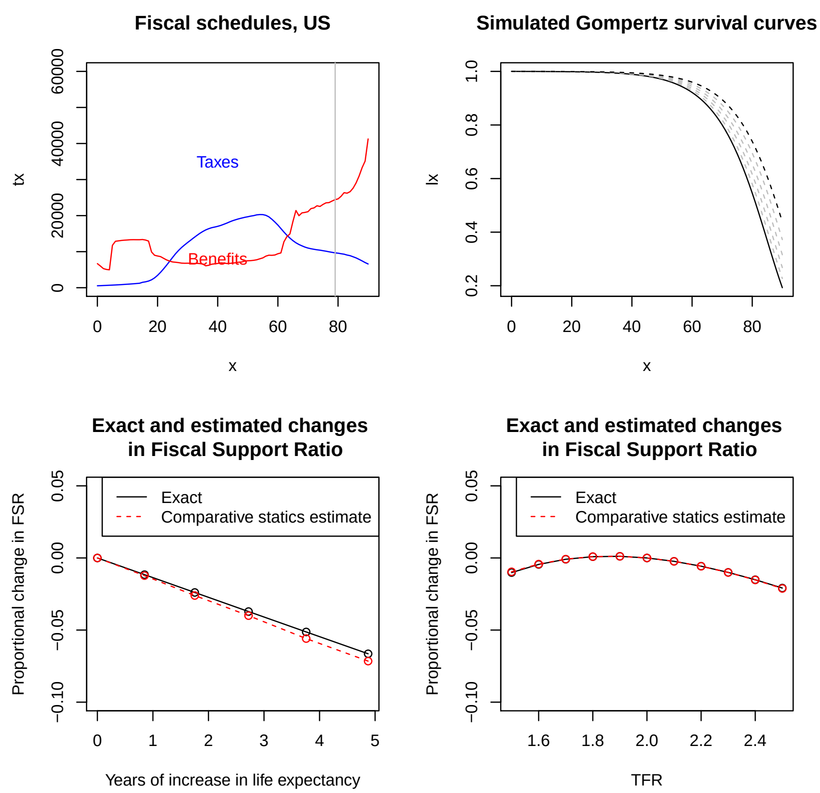

Figure 1: Simulating the impact of changes in fertility and mortality on the U.S. Fiscal Support Ratio using observed NTA schedules of taxes and benefits

The exact and estimated changes in the FSR are shown in the lower panels. The figure reveals two important features. First, it shows that the approximation used in equation (7) is quite accurate, even if not as exact as is the case with changes in the fertility rate. The error is hard to detect with a life expectancy change of 1 year and detectable but small even for changes of five years. Second, we see that the magnitude of change in the

Fiscal Support Ratio is many times larger for longevity changes than fertility changes. Whereas a life expectancy increase of 5 years reduces the FSR by about 8 percent. A TFR change of plus or minus a half a child reduces the FSR by only about 1 percent.

5 Extensions

5.1 Dependency Ratios

The comparative statics approach can be applied traditional metrics like dependency ratios comparing the numbers of people in working and dependent age-groups. In these cases, one can replace the observed empirical curves for $t(x)$ and $b(x)$ with the binary distinction, 0/1, at each age. For example, for old-age dependency ratio, with age boundaries 15 and 65, t(x) = 1 for $15 \leq x \leq 65$ and $t(x) = 0$ for $x \geq 65$; and $b(x) = 1$ for the old ages and zero for the working ages. For the case, when $\bar{x}$ (or e_0) is an age classified as "old", then equation (7) takes the form

$$\frac{d \log OADR}{de_0} \approx \frac{1}{T_{65}},$$

where T_{65} is the life table quantity for the number of person years lived age 65 and over.

Similar relationships can be developed for the YDR and $OADR$.

5.2 Non-stationary populations

[to do]

6 Concluding remarks

In this note, we have developed a comparative static calculation for increases in longevity to complement and contrast the classic relationship for changes in stable population growth rates. The two comparative statics calculations allow a full account of the long-term consequences of changes in longevity and/or fertility on measures like the fiscal support ratio. The fiscal support ratio is expected to decline from about 1.0 today to values in 2060 of 0.8 in moderate-fertility France and 0.7 in low-fertility Germany. Our comparative statics calculations were able to capture these effects, predicting about a 20 percent decline due to longevity alone, with another 10 percent decline for the half-a-child lower fertility rate in Germany.

Considering fixed age-schedules of characteristics like benefits and taxes, this kind of analysis makes it clear that the impact of expected longevity improvement on public finances is potentially much larger than the consequences of moderate changes in fertility. The kind of stable population growth rate that it would take to account for 10 years of longevity improvement would have to increase the growth rate by some 2 percent per year, the equivalent of an increase from a TFR of 2 to something more like 3.5 children per woman (the peak of the baby boom in the United States).

The caveat of the preceding paragraph of "fixed age-schedules" is crucial. The consensus view on how to adapt to population aging is to slowly change the age-schedules of work and retirement. Such adaptation is made easier by the improving health of older people that has – so far – been associated longer life. In essence, we rescale our work-lives in step with the rescaling of life itself.

The age-structure effect of fertility change is taken into account by the comparative statics approach, but other important effects are left out. The tendency for human capital investments to increase when fertility declines (Becker, Lee new results) are an important mechanism by which declining labor force size can be offset by increased productivity from higher per capita investments in human capital. Similarly, as we know from neo-classical growth theory (Solow), slower population growth will, all other things equal, increase capital-labor ratios, also increasing worker productivity.

The feedbacks in terms of individual behavior and public policy, generated by fertility and mortality change, and by population aging, are all important factors to take into account before drawing conclusions from the kind of comparative statics analysis given here.

From this perspective, we see that although the low fertility might have a smaller effect on finances than longer life that a lower birth rate lacks the same kind of automatic policy response associated with increasing health and longevity. Why should people be able (or want) to work longer simply because they (or others) are having fewer children? The extreme case of some of the former Soviet republics is a cautionary tale. In countries like the Ukraine, life expectancy is not improving much, but fertility has fallen dramatically. Here, the population will have to adapt to a larger share of the elderly, without a concurrent increase in healthiness.

On the other hand, low or negative rates of population growth can also have effects that go beyond age-structure including the deepening of physical capital and the potential for increased investments in human capital.

These kind of considerations make it clear that some caution must be exercised in making quantitative comparisons about the "importance" of different demographic factors in population ageing. Comparative statics calculations provide useful information about the demographic causes of population aging, but the relative magnitudes obtained from such calculations should be thought about carefully. For example, one could argue that even though increasing longevity appears to be a quantitatively larger challenge, it is one with a ready-made, comparably-sized solution – the stretching out of the economic life cycle. Likewise, fiscal shortfalls caused by fertility decline may be smaller in scale but, at first consideration, do not have such an automatic adjustment mechanism.

7 References

[to do]