Displaying Nonlinear Age-Period-Cohort Patterns on Lexis Plots

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Abstract

The identification of age-period-cohort (APC) patterns on vital rate changes over time is of great importance for the understanding of demographic phenomenon. Graphical analyses are often regarded as more transparent than statistical modeling in the identification of APC effects. The current paper proposes a Lexis plot for the analysis of irregularities in vital rates that are indicative of nonlinear APC effects. This visual display offers a higher level of flexibility than mathematical models and other Lexis plots. In one single visualization we combine the dynamics of the location, magnitude and spread of temporal effects for multiple populations or demographic phenomenon. Using vital rates, we provide three examples analyzing APC nonlinear effects on different demographic phenomenon. We display patterns of modal cohort of baby boomers' excess drug-related mortality by racial/ethnic group in the US; modal age of young-adult excess mortality by country; and modal age of fertility over cohorts across populations.

Introduction

It has long been recognized that populations change along the three dimensions of age, period (typically calendar year) and cohort (typically year of birth) (Caselli and Vallin 2005; Keiding 2011). Whether it is possible to independently isolate these age, period, and cohort effects on the temporal change in population phenomena has been subject to strong debates since the first half of the 20th century (Murphy 2010), which recently reached a new peak of heated discussions with the set of methods proposed by Yang and colleagues (Bell and Jones 2013; Luo 2013; Reither et al. 2015; Yang and Land 2013). At the center of the debate is the identification problem given the perfect linear dependence between these three dimensions (age = period - cohort), which make it impossible to estimate a unique solution without the imposition of additional constraints.

Given this limitation, graphical analyses are often regarded as more transparent than statistical modeling in the identification of age, period, and cohort effects (Murphy 2010; Preston and Wang 2006; Willets 2004). Consistent to this idea, the hand drawn contours indicating cohort mortality improvement patterns in the work of Kermack, McKendrick, and McKinlay (1934), has been recognized as a pioneering work in demonstrating that mortality change was following the birth cohort dimension (Finch and Crimmins 2004; Hobcraft, Menken, and Preston 1982; Murphy 2010; Preston and Wang 2006). Nevertheless, whether the identification of such patterns could also be interpreted as 'cohort effects' (i.e. whether improvements in mortality were the consequence of period-based improvements or of a better performance of younger birth cohorts), was and remains contentious, as with the case of APC statistical models; see Murphy (2010) for an interesting review.

Detrended effects

Less polemic and more effective than the approaches inquiring about dominant patterns of change, have been the analyses in which the attention is focused instead on identifying independent second difference parameter estimates (also known as non-linear fluctuations) of each of the three temporal dimensions (Clayton and Schifflers 1987; Holford 1983). For example, such an analysis might seek to identify a systematic fluctuation that follows a cohort, which is independent of changes over age and period dimensions. These fluctuations could occur above (humps) and below (valleys) the linear trends. If such irregularities followed horizontal, vertical, or diagonal trends on a Lexis surface, it would be indicative of age, period, or cohort effects respectively.

Some alternative arithmetic and statistical models have been proposed to allow the extraction of these divergences from the trends, such as the *Median Polish* technique (Tukey 1977) or the *Detrended Age-Period-Cohort models* (Carstensen 2007). However, a considerable limitation to these methods, which consist of an estimation of constant effects that are invariable during the whole length of time, is their inflexibility. In particular there are two undesirable consequences; first, the methods do not allow the size of the effect to vary over time/age, and second, they attribute the highest or lowest effect to a fixed temporal dimension, labeling it permanently as advantaged or disadvantaged. An alternative model (Chauvel 2013) allows the estimation of an hysteresis value that indicates whether the magnitude of the divergence from the trend is increasing or decreasing over time, but still, it is a fixed measure that does not allow for changes in the hump/valley location over time/age.

Graphical tools for identifying non-linear fluctuations

Several graphical tools have been proposed for uncovering these patterns of systematic divergences from the linear trends, focusing mostly in the analysis of mortality. The plot of the derivative of the smoothed rates of change by age and calendar year is an effective tool for unveiling the dynamic of demographic phenomenon over time, and useful for uncovering patterns of systematic divergence in any of the three temporal dimensions from the linear trends (age, period or cohort effects). Indicative Lexis diagrams (Willets 2004) and colored Lexis surfaces (Rau et al. 2008, 2012, 2018; Richards, Kirkby, and Currie 2006) of these derivatives have been proposed as effective visualization techniques to identify the presence of such patterns in Lexis plots.

In contrast to the detrended APC models, these plots of rate derivatives are considerably more flexible in depicting the non-linear fluctuations, allowing the shapes of humps and valleys to move freely through the Lexis surfaces, and depicting the pattern with a higher fidelity to the observed data. Additionally, these plots have the great advantage of depicting general patterns modulating the changing phenomena over a broad age and time surface. In a single image it is possible to identify several irregularities that would be indicative of age, period, and cohort effects on the dynamic of a specific phenomenon of a population.

However, whereas these plots are very useful for the analysis of the temporal pattern of a single phenomenon in a single population, when comparisons across several phenomenon in a single population, or a single phenomenon across several populations are desired, it is compulsory to construct as many surfaces as phenomenon/populations are proposed to parallel. Besides the high requirement of space, this option also has the limitation that contrasting patterns across surfaces is visually difficult, even when surfaces are facetted.

The proposed visualization

The current paper follows in the tradition of the aforementioned literature in its attempt to visually depict and contrast the nonlinear changes in demographic phenomenon over the age, period, and cohort dimensions. Specifically, we demonstrate the value of a broad picture comparison of how the location of the maximal ridge or hump of a phenomenon is changing over time. This ridge could be anything from the location of maximum excess mortality by age and cause of death to the mode of age-specific fertility. In the former case, the visualization enables the comparison of the temporal patterns for many phenomenon (causes of death) in one population on one plot. In the latter case, the interest is in contrasting the temporal patterning of a single phenomenon (age-specific fertility schedules) across a variety of populations or subgroups. While the simplest visualization depicts the changing location of these demographic phenomenon, the use of color, size and intensity allows for more information about the densities to be depicted, including, for instance spread or magnitude values.

It is important to notice that when the emphasis is put on the maximum/minimum point of the hump over time, other local maximums/minimums would be neglected in the case of multimodal distributions. This information could be of great importance, and alternative plots could depict the dynamic of multiple modes over time. Nevertheless, this choice depends on the context of the research question, and should strategically consider the tradeoff between visual complexity and clarity of the visualization (Munzner 2014).

Although the plots proposed here contribute with additional information not available in APC statistical models, they should be considered as complementary. Some estimable functions from statistical APC models are of great value and are not obtainable by graphical analysis, such as detrended APC effects and their confidence intervals.

In what follows, we describe the procedure to construct the comparative Lexis plot of these demographic ridges and apply the procedure to three empirical examples: (1) the excess mortality of baby boomer cohorts in the United States from drug-related causes of death across racial/ethnic populations, (2) the stability of the age location of the young-adult mortality hump across countries, and (3) the temporal patterning of age-specific cohort fertility peaks.

Construction of the plot

The construction of the Lexis plot involves four main steps:

1. *Definition of data resolution*

   For this kind of analysis we suggest to use the finest possible resolution of data. The smaller the grid in the Lexis surface the clearer would be the depiction of the temporal dynamics of these demographic ridges.

2. *Identification of the ridges and temporal section frame of interest*

   As mentioned in the introduction, there are pre-existing visualization tools such as the Lexis surface of change in vital rates over age and period that allow demographic ridges and valleys to be identified. The rate of change in mortality and fertility is generally noisy, particularly over ages and causes where small changes in the event can lead to large changes in the rate of change. Thus we suggest to smooth the underlying data over ages and years in order to reduce random fluctuation. A number of alternatives are available for smoothing. In the examples that follow, we use a two-dimensional non-parametric smoothing technique. Specifically, we assume that our events are Poisson-distributed and smooth the data with P-splines, using the *MortalitySmooth* R package developed by Camarda (2012). The smoothing parameters can be optimized according to AIC or BIC.

   After smoothing the vital rates, the next step is to estimate and plot their change. Variations over age and calendar time are two complementary perspectives to visually identify non-linear trends (ridges/valleys) on a Lexis surface. When looking at rate changes over calendar-year (horizontal changes in the Lexis diagram) we are identifying within-age category divergence in the change and unveiling period and cohort patterns that are present independent of age effects on the vital rate of interest. This perspective is useful in the absence of major abrupt changes over the period, for instance for some endogenous causes of mortality (neoplasms, cardiovascular diseases), or for some behavioral causes with temporal stability, meaning that changes in the event are moving smoothly across time and not subject to strong year-to-year fluctuation (alcohol abuse in the US).

Analogously, when looking at divergence related to rate changes over age (vertical changes in the Lexis diagram), these are evaluated within the same calendar-year, so that age and cohort divergence from the trend are independent of period effects. This perspective of change would be more useful for phenomena with strong and frequent period fluctuations, as in the case of seasonal epidemics (i.e. influenza) or behavioral causes with sudden and strong variations over time (i.e. HIV/AIDS and drug overdose epidemics).

Once the surface ridges and valleys for the analysis have been identified, their temporal position in the Lexis diagram determines the dimensions and location of the temporal frame of interest. This framing is important since other “irregularities” located outside of this temporal section would be a potential source of noise for the analysis.

3. *Estimation of humps/valleys features*

Remund and colleagues (2017) proposed three attributes of interest in describing the young adult mortality hump with examples of summary indices to measure them; magnitude (loss in life expectancy, years of life lost, and death counts), location (mode, mean, and median), and spread (standard deviation and quantile). These dimensions and indices could be translated and enriched to analyze other demographic phenomenon.

The only requirement of the comparative Lexis ridge plot is a location measure (i.e. mode, mean, etc.). Other dimensions, i.e. magnitude and spread, are optional and complementary measures that could enrich the analysis and comparison but are not strictly necessary for the construction of a basic version of the plot.

In cases where the deviance from the trend is also a local or absolute maximum or minimum, the mode of ridges and valleys could be easily obtainable through simply extracting the age/period coordinates of the maximum or minimum smoothed values within the temporal section frame. However, for cases when these irregularities are not local maximum or minimum the estimation of their respective location, magnitude, and spread require additional information pertaining to the excess or depths of vital rate estimates.

Several methods are available to estimate a baseline of mortality from which it would be possible to estimate these excesses or depths in vital rates. These include interpolation techniques (Camarda 2012), decomposition of the irregularities (Remund, Camarda, and Riffe 2018), or simply detrending the smoothed vital rates over the selected perspective of change (i.e. over age or over period). There is no ideal generic method since each demographic phenomenon and research question has a specific underlying hypothesis that should be accounted for. The location, magnitude, and spread of the pattern irregularities can be estimated from these excess or depths in vital rates. It is worth noting that all three features could be estimated under age, period or cohort perspectives, according to the temporal dimension under interest.

Magnitude and spread measures could be estimated for the pattern irregularity as a whole, which would be a fixed measure over age and time. However, if a visualization with complete information about changes to the shape of the demographic phenomenon over age and time is desired, magnitude and spread measures should be estimated individually for each location point.

4. *Translate hump and valley measures to visual properties of the plot*

   The population or the subtype of demographic phenomenon to compare in the Lexis surface is a categorical value that should be translated to color, preferably using a color blind safe palette, or to different point shapes for the case of a black and white printout[1]. Location measures should be translated to the age and period coordinates in the Lexis diagram. These point coordinates should preferably not be unified by lines in order not to fuse unrelated humps or valleys.

   The magnitude measure should be standardized and inversely translated into a transparency level. And finally, spread measure should be standardized and translated to shape size.

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[1] When using different point shapes, it should be guaranteed equal areas to equal values of spread across the different point shapes employed. Most of the time this is not the case and a correction factor must be applied to adjust the proportionality of areas across shapes.

Empirical Application

For illustrative purposes we applied the above described procedure to construct Lexis plots of non-linear irregularities to three demographic outcomes with different temporal perspectives: a) Boomers' drug related excess mortality comparing several ethnic groups in the US (hump across cohorts, along periods); b) young adult excess mortality, comparing several countries (hump across ages, along periods); and c) cohort fertility rates (hump across ages, along cohorts). In this section we describe some details about the estimation of the location, magnitude and spread features, as well as their translation to a Lexis plot displaying the attributes for each case.

a. Boomers' drug-related excess mortality

Previous exploratory and descriptive analyses (Acosta et al. 2017; Miech, Koester, and Dorsey-Holliman 2011) have suggested that Baby Boomers in the US have experienced a disproportionate susceptibility to drug-related mortality throughout their life course. Thus, for this specific case, we are interested to compare the attributes (position, magnitude, and spread) of *cohort mortality humps of drug-related mortality* for several ethnic groups of US males in a single Lexis plot.

We use mortality data by single calendar year, single year of age, sex, race, ethnic group, and cause of death, from the US National Vital Statistics System (NVSS 2018) from 1980 to 2016. This data allows us to use single-year resolution in the estimation and visualization of non-linear cohort effects. Here we define drug-related mortality as death involving drug use and registered within categories of accidental and undetermined intent overdoses, and within mental and behavioral related causes of death.

To identify cohort mortality humps we constructed Lexis surfaces of rates of smoothed mortality change over period and over age for each ethnic/race group of males in the US. We smoothed the underlying data (**Fig. 1A** for the Hispanic ethnic group as example) using the R package *MortalitySmooth*, with the smoothing parameters optimized by AIC.

When plotting mortality change over age (**Fig. 1B**), it is evident that independent of strong period variation, there is a diagonal mortality hump (indicative of a disadvantaged cohort), whose peak follows the cohorts born between 1950 and 1960 throughout their life course. These disadvantaged cohorts are surrounded by two parallel valleys, suggesting two sets of advantaged cohorts, one born around 1940 and the second one born at the beginning of the 1970s, respectively. The trajectory of the humps are not completely straight and their magnitude and spread are not constant over time. These patterns vary considerably across ethnic/race groups (not showed here).

Lexis surfaces of drug-related mortality for Hispanic males in the US

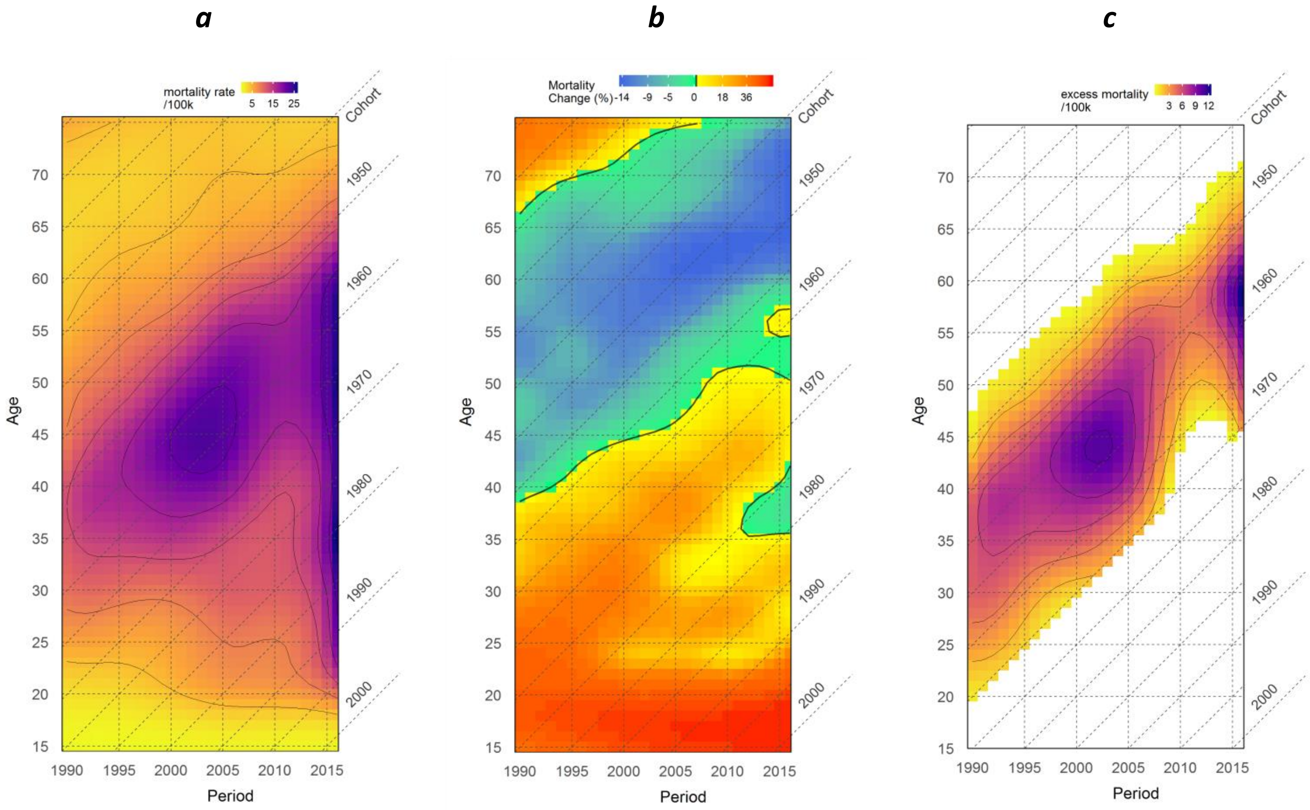

Figure 1. *Lexis surfaces of drug-related mortality rates (a), drug-related mortality change over age, with a yellow to red scale indicating mortality increases and a green to blue scale for mortality decreases (b), and Boomers drug-related mortality excess (c) for Hispanic males in the US during 1990-2016, ages 15-75. Note: In this case, the mortality change over age plot (b) should be read vertically as change of mortality for age x over age x-1, in the same year period. The black contour line depicts zero change in mortality, which indicates a local minimum or maximum death rate in a given year period.*

In order to compute the features of the humps, we must estimate the excess mortality for the identified disadvantaged cohorts of each racial/ethnic group. A mortality baseline is obtained through the interpolation option available in the R-package *MortalitySmooth*. We exclude deaths pertaining to the identified affected birth cohorts born between 1940 and 1970, and interpolate the surface with these cohorts removed. The cohort excess mortality is defined as the difference between the smoothed observed mortality rates and the mortality baseline (i.e. the predicted rates had mortality developed according to trends observed before and after these disadvantaged cohorts). From these excess mortality estimates (**Fig. 1C**) we are able to extract information about the dynamic over time of the location, magnitude and spread of the hump, during the period 1990-2016.

In **Fig. 2** we plotted the following attributes of the excess boomer drug-related mortality on a Lexis diagram: (1-category) each observed racial/ethnic group was identified by a distinct color, or by a different point shape for the case of a black and white printout (Figure A1), (2-location) for each calendar year we plotted the cohort-location of the maximum relative risk in drug-related mortality, (3-magnitude) the relative risk compared to the baseline value for each point was translated into opacity, (4-spread) and the standard deviation of the hump in relative risk for each period was translated into point size.

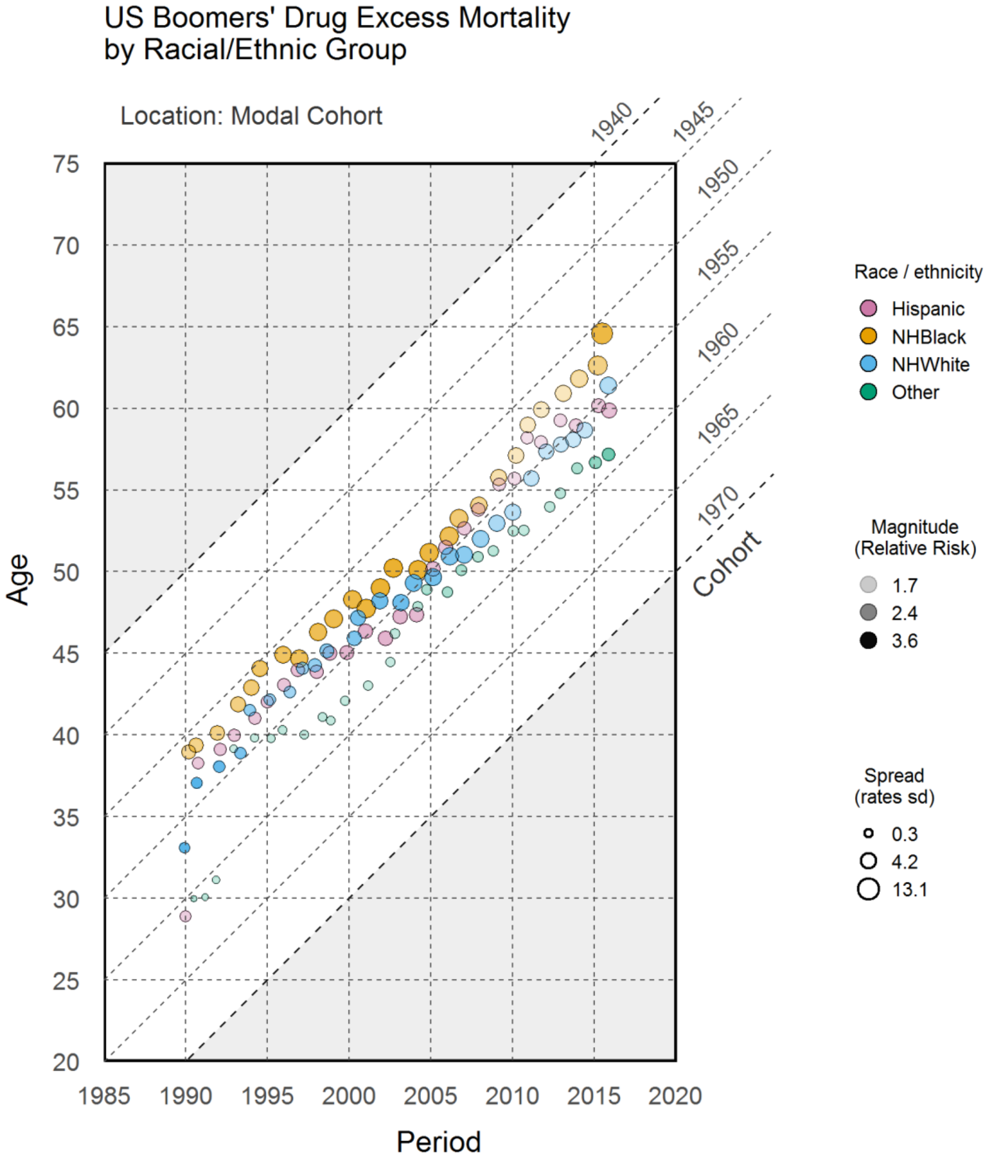

Figure 2. *Lexis plot of drug-related excess mortality features by racial/ethnic group for males in the US. Note: The greyed out upper and lower triangles indicate the cohorts that were used for the estimation of the mortality baseline (i.e. before birth cohort 1940 and after birth cohort 1970), and thus are constrained to have no excess mortality.*

In this case, not only were we able to extract and summarize the differences in location from 4 different plots into a single visualization, but we could also contrast additional outputs to the rate of mortality improvement plots, such as the magnitude (relative risk) and spread (standard deviation) of the mortality hump compared to the interpolated or expected values. In doing so, the clear identification of disadvantaged cohorts for all ethnic-race groups was evident by the alignment of the points along a diagonal cohort, while the degree of cohort disadvantage being stronger among non-Hispanic Whites and non-Hispanic Blacks compared to other race groups was seen by the darker tones among the former groups. Note that we picked a cause of death with particularly strong cohort patterning. For other causes of death, for instance cerebrovascular diseases, such nonlinear excess mortality risks would not follow cohort patterns, and as a result, the depicted points would not fall along a diagonal line.

b. Young adult mortality hump

Young adult excess mortality, also known as the accident or young adult mortality hump is a well-known feature of the age structure of most mortality regimes, particularly among males. Goldstein (2011) presented a plot of the hump peaks over time for several countries, and Remund and colleagues (2018) decomposed the contributions by cause of death and analyzed several features of the hump in the US. Here we used mortality data from the Human Mortality Database (HMD 2018) for some countries in order to compare not only the ridges of the hump, but also the additional features of their change in magnitude and spread over time and across different countries. Similar to the method employed by Goldstein, as well for the case of the Boomers' example, we defined the hump as the difference between the cross-sectional period smoothed mortality rates and an interpolated mortality baseline between ages 10 and 40.

We compared Spain, Russia, Taiwan and the US during the period 1965-2016. In this case, **Fig. 3** shows the variation over time of the modal age of the hump, its magnitude measured by excess death rates, and the spread of the hump measured by standard deviation.

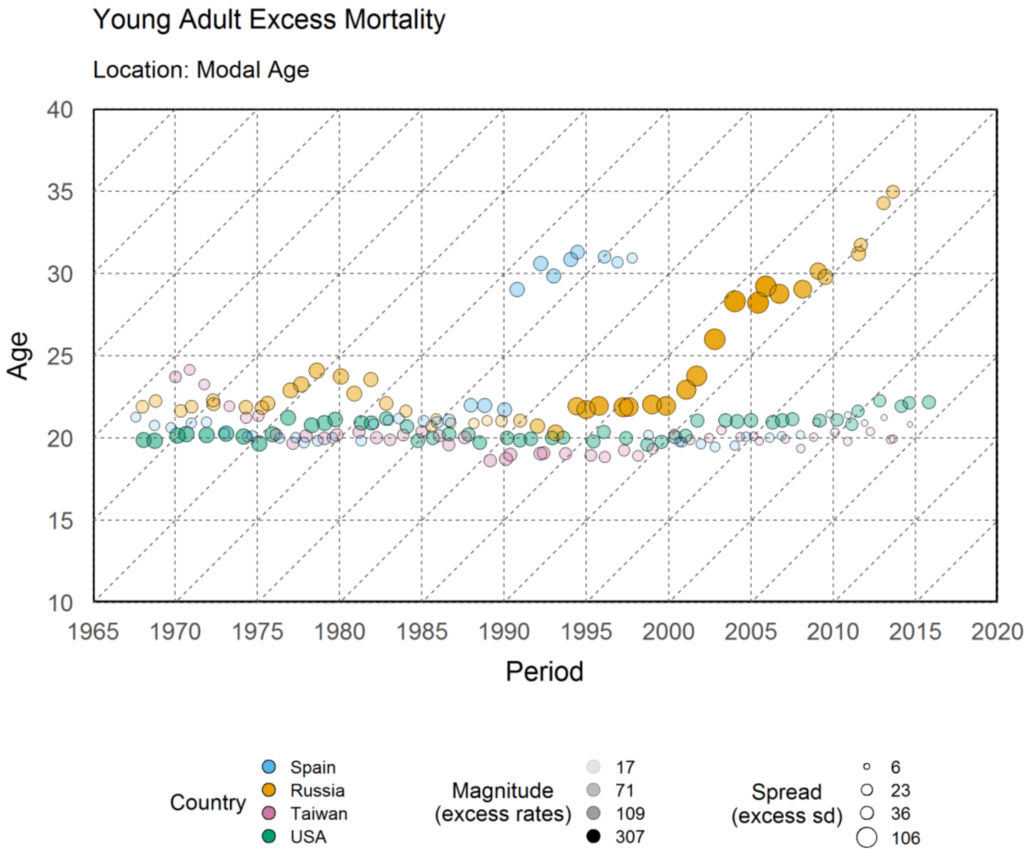

Figure 3. *Lexis plot of young adult excess mortality features by country for males*

c. Cohort fertility rate

As a final example to show applications beyond mortality, we plotted some aspects of fertility behavior along cohorts. **Fig. 4** depicts age-specific fertility outcomes of the cohorts born between 1905 and 1985 in Spain, Sweden, and the US. Cohort age-specific fertility rates (ASFR) were obtained from the Human Fertility Database (HFD 2018).

In this case we plotted the modal age of fertility for each cohort, the ASFR pertaining to this mode, and the standard deviation before that age. The standard deviation before the mode was used in order to consider more recent cohorts who are likely to have reached the modal age in fertility, but have not completed their childbearing. As previously mentioned, any number of indices could be used to capture the three dimensions of magnitude, intensity, and spread.

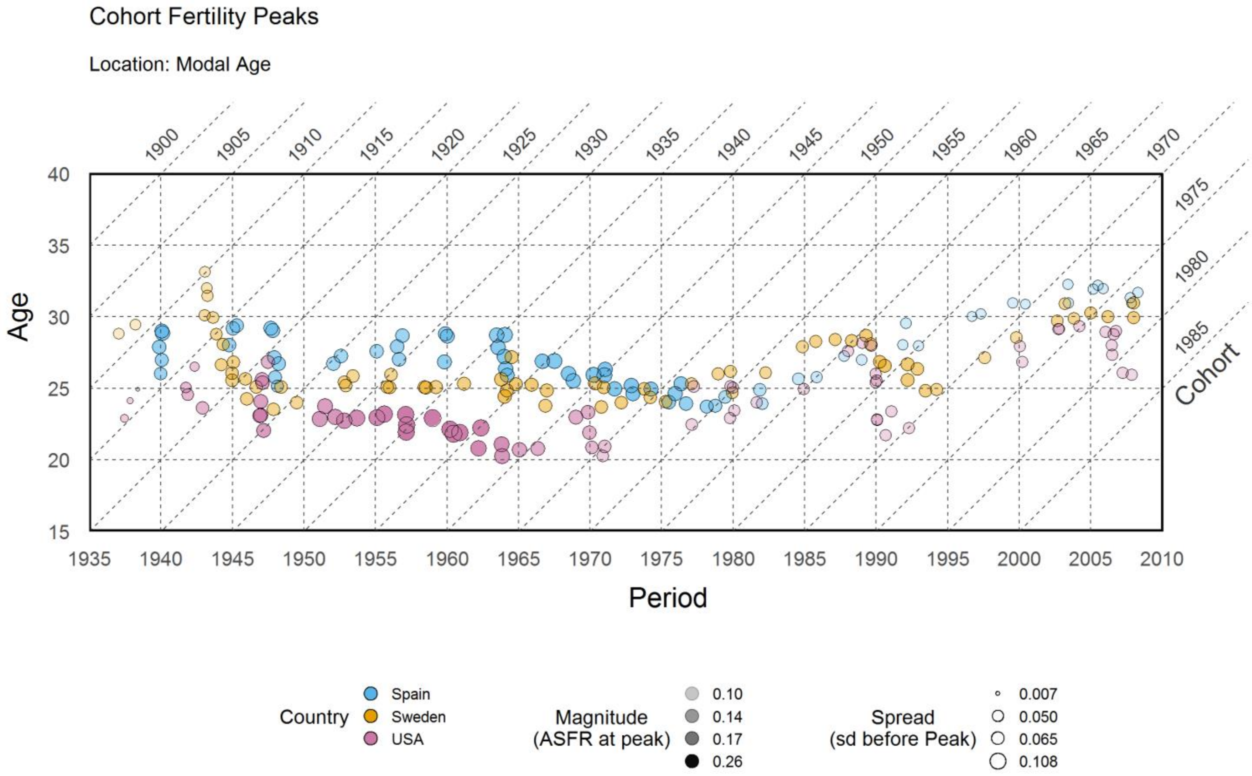

Figure 4. *Lexis plot of cohort fertility rates by country*

Conclusion

We discussed the advantages of using visualization tools on Lexis diagrams for the analyses of age-period-cohort non-linear effects of vital rates compared to mathematical models, as well as some of their limitations. We proposed a Lexis diagram to enrich the analysis of irregularities in vital rates that are indicative of age, period, or cohort effects. We claim that unlike mathematical models and other Lexis plots, this visual display offers a higher level of flexibility, because it allows us not only to depict the dynamic of the location, magnitude, and spread of temporal effects over time together, but also to contrast different populations or subtypes of demographic phenomenon in a single visualization. We outlined the process to construct Lexis plots for the analysis of age, period, and cohort effects. Using vital rates, we provided some examples analyzing cohort effects on drug related mortality by racial/ethnic groups within the US, age effects on young adult mortality in Russia, Taiwan, Spain, and the US, and age/period effects on fertility in Spain, Sweden, and the US.

Appendix

Black-and-white printout of the Lexis plots

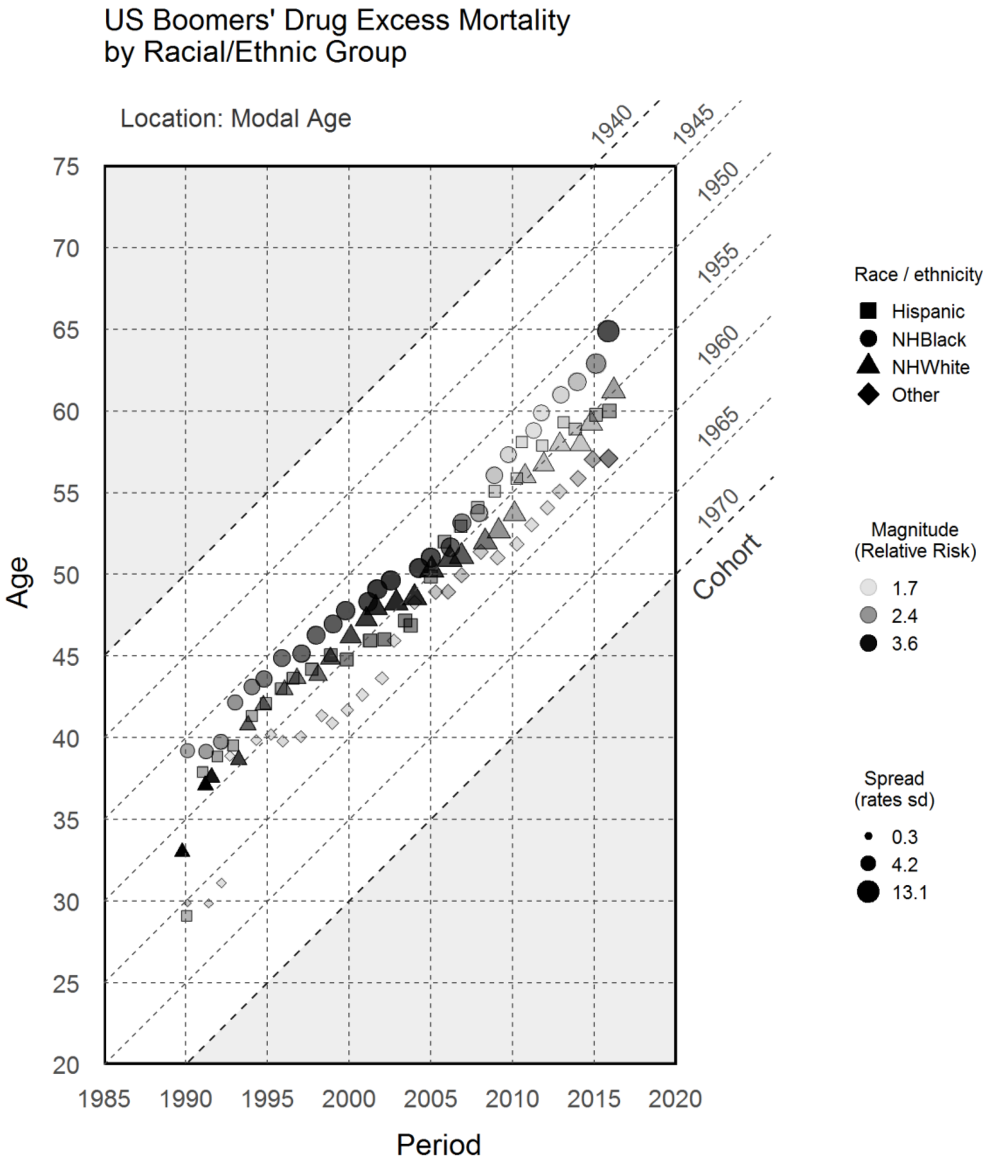

Figure A1. *B&W Lexis plot of drug-related excess mortality features by racial/ethnic group for males in the US*

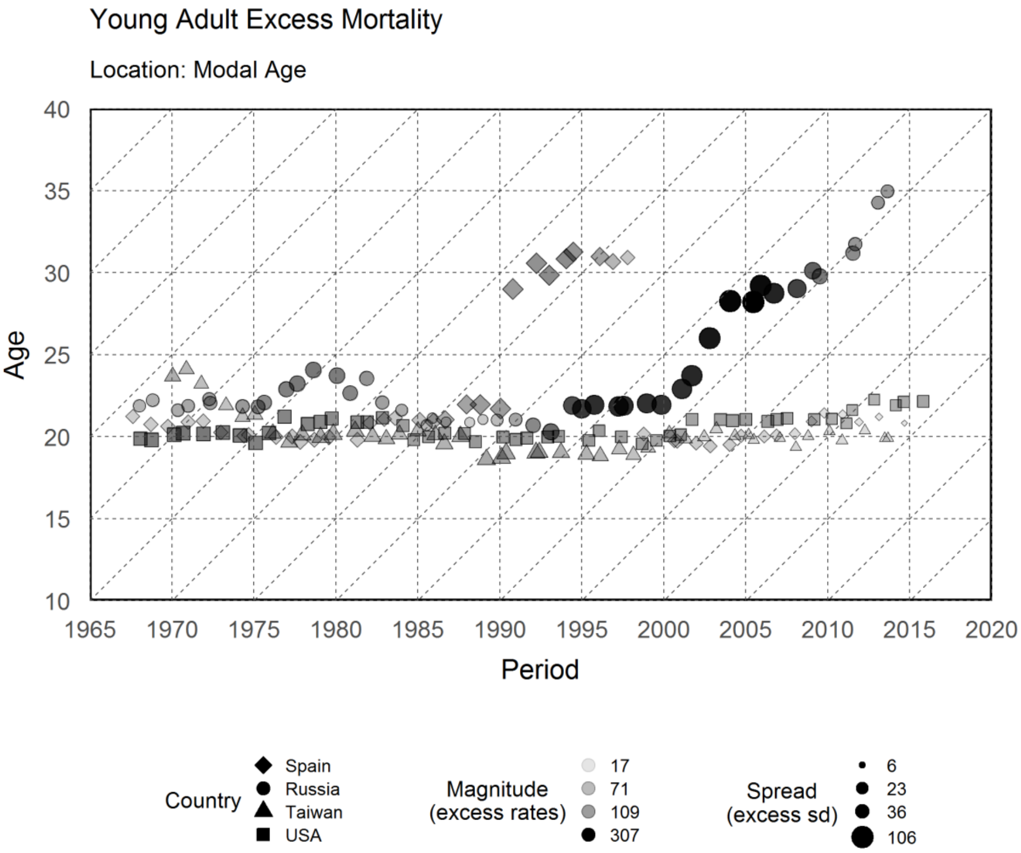

Figure A2. *B&W Lexis plot of young adult excess mortality features by country for males*

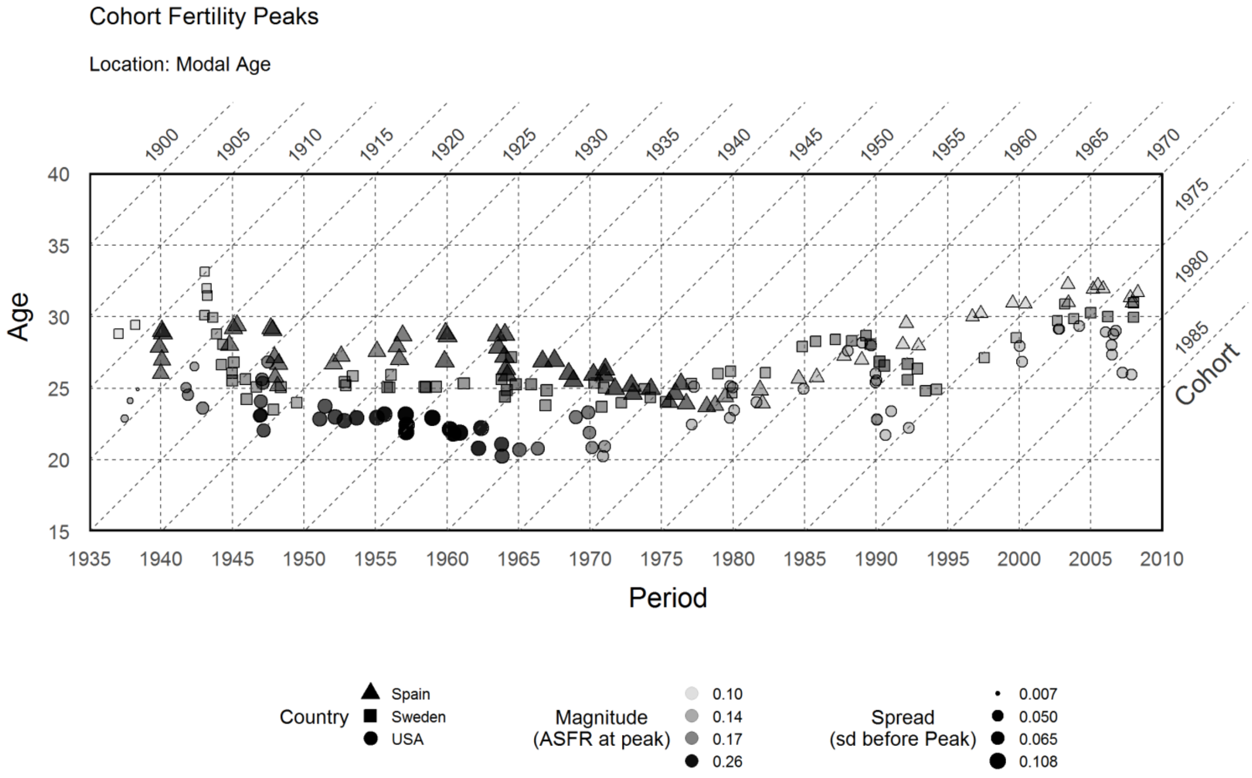

Figure A3. *B&W Lexis plot of cohort fertility rates by country*

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