Joint Modeling of Multivariate Survival Data with An Application to Marital Dissolution and Retirement

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Abstract

The Cox proportional hazards model has been pervasively used in many social science areas to examine the effects of covariates on timing to an event. The standard Cox model is intended to study univariate survival data where there is a singular event of interest which can only be experienced once. However, we may additionally wish to explore a number of other complexities that are prevalent in survival data. For example, an individual may experience events of the same type more than once or may experience multiple types of events. This study introduces innovations in recurrent (repeatable) event analysis, jointly modeling several endogenous survival processes. As an example and an application, we simultaneously model two types of recurrent events in the presence of a dependent terminal event. This model not only correctly handles different types of recurrent events but also explicitly estimates the direction and magnitude of relationships between recurrences and survival. The paper concludes with an example of the model to examine how the timing of retirement is associated with the risk of union dissolution. The theoretical discussions and empirical analyses suggest that the multivariate joint models have much to offer to a wide variety of substantive research areas.

1 INTRODUCTION

In the past several decades, a growing recognition of survival analysis has been propelled by the availability of longitudinal or retrospective surveys as well as advances in statistical science. The Cox proportional hazards model (Cox 1972), a widely used procedure in survival analysis, was originally developed by Cox studying the occurrence and timing of terminal events. The issues of timing and sequence of life events are highly relevant for social science researchers and these methods which incorporate covariate effects on the risk of observing an event are used, for example, to model union formation and dissolution (Pessin 2017; Schneider 2011; Schimmele and Wu 2016), fertility behavior (Axinn, Dirgha and Smith-Greenaway 2017; Balbo and Barban 2014), job mobility (Blossfeld and Drobnic 2001; Blossfeld, Hamerle and Mayer 2014), to name a few.

The standard Cox model applies to univariate survival data when there is a singular event of interest and the event can only be experienced once. In addition, event times are assumed to be statistically independent (Ezell, Land and Cohen 2003; Grambsch and Therneau 2000). Grappled with increasingly dynamic and heterogeneous life trajectories, social science researchers may additionally wish to explore a number of relevant issues found in multivariate survival data. For example, how does one employ survival analysis to study repeatable events (a singular event type happening more than once) and different types of correlated events?

Unlike many biostatistical studies, in which the event under study may occur only once (e.g., the death of a patient), the majority of events in social sciences are repeatable, such as marriages, divorces, and employments. Recurrent (repeatable) data arise when the same type of event can occur to a subject multiple times. The standard Cox model is not suitable because it assumes a subject is not at risk any longer after the subject experienced the first event. In the analysis of recurrent data, all subjects are at risk of new events as long as they are not censored or have experienced a terminal event, like death (Commenges and Jacqmin-Gadda 2015). In addition, when a subject experiences the same type of event multiple times, the timing of these events is likely to be correlated within subject. Failing to account for the repeatability of the event is tantamount to imposing an independence assumption on the occurrence of the events, which often leads to biased estimation (Box-Steffensmeier and Jones 2004).

Besides repeatable events, there is considerable interest in studying the timing and sequencing of interdependent life events. The outcome of one life event can influence the occurrence of another life event. For example, Lillard's (1993) research shows that the risk of marital conception is associated with a decreased risk of marital disruption. On the one hand, it is well known that children affect the chances that their parents will divorce (Lillard and Waite 1993). Having children together raises the costs of divorce and increases the gains from marriage, leading to greater marital stability among couples with children (Lillard and Waite 1993). On the other hand, it is also plausible that people take into account anticipated changes in marriage duration in their childbearing decision (Lillard 1993). Therefore, the decision about childbearing and the decision to remain in marriage (or end a marriage) may be subject to shared unobserved factors (Steele 2011). Childbearing outcomes may be endogenous with respect to marital disruption. Estimation of the impact of anticipated events on current event transitions is challenging as these anticipation factors are unobserved (Ermisch and Steele 2016). It appears that social scientists have yet to develop a standard approach for tackling correlation between endogenous survival processes.

Fortunately, in recent years joint modeling of several survival processes has received considerable attention in statistical research because it can be used to address some interesting scientific questions that could not be answered before, such as the impact of anticipated events on current transitions as well as quantifying the relationships among them. Separate modeling of each survival process may not fully reveal potential mechanisms and can produce misleading results. Appropriate statistical methods are needed to utilize the richness of these data in order to identify potential relationships between different endogenous survival processes.

Given the ubiquitous repeatable events in social science research, as well as longstanding interests in examining interdependence of life events, we introduce a new dynamic approach to model survival data, which extends the Cox model by incorporating repeated and/or simultaneous events. Most of existing studies using this type of model were conducted in biomedical fields. As a result, these methodologically advanced techniques are relatively unfamiliar to social science researchers. Considering the complexity of dynamic life events, we would like to extend social scientists' statistical toolkit beyond the standard Cox model by taking advantage of recent developments in multivariate survival models.

Specifically, in this paper we introduce a multivariate joint frailty model for two types of recurrent events in the presence of a dependent terminal event, with right censored survival data. Our application is based on retrospective data from the 2007 Canadian General Social Survey, Cycle 21 (GSS-21), conducted by Statistics Canada. We treat two types of marital dissolution - widowhood and divorce - as repeatable events and discuss how to appropriately handle them using shared frailty models. We aim to assess how the hazards of recurrent events (widowhood and divorce) would impact the hazard of terminal event (retirement) and at the same time explicitly estimate the strength of associations among these three survival processes. Widowhood, divorce, and retirement transitions are all specified in separate equations but are estimated in a joint maximum penalized likelihood procedure (see Mazroui et al. 2013). This allows us to analyze the dependencies of the transitions explicitly, controlling for the potential endogeneity of each transition with respect to all the others.

We begin with a review of recurrent events analysis and discuss the use of a shared frailty model to analyze recurrent events. Our attention then turns to an introduction to multivariate joint frailty models and an estimation method in section 3. Then we apply multivariate joint frailty models to an analysis of a Canadian national survey with observations of two recurrent events (divorce and widowhood) and terminal event (retirement) in section 4. The final section provides conclusions and discussions.

2 LITERATURE REVIEW

2.1 Cox Model

The Cox model analyzes effects of covariates on the hazard rate. Let $h(t;\mathbf{Z})$ be the hazard rate at time t for an individual with risk vector $\mathbf{Z}$. A Cox proportional hazards model is defined as follows:

$$h(t;\mathbf{Z}) = h_0(t)exp(\beta^T\mathbf{Z}) \tag{1}$$

where $h_0(t)$ is the baseline hazard rate or reference value, β is a parametric vector, $exp(\beta^T \mathbf{Z})$ is the relative risk, a proportionate increase or reduction in risk. The model is called semi-parametric because it does not make specific assumptions about the baseline hazard function.

Parameter estimates in the Cox model are obtained by maximizing the partial likelihood as opposed to the full likelihood. The partial likelihood allows estimation of covariates without making any assumption on baseline hazard. This is a key reason for the popularity of the Cox model. The partial likelihood function is derived by taking the product of the conditional probability of a failure at each time, given the number of subjects that are in the risk set at that time (Cox 1972; Kleinbaum and Klein 2012; Klein and Moeschberger 2005).

2.2 Recurrent (Repeatable) Events

In many studies, individuals may undergo the same type of event several times during the follow-up period (see, e.g., Cook and Lawless 2007). Common examples of recurrent events include heart failure hospitalization, asthma attack, marriage, divorce, and unemployment.

Analyzing repeatable events of the same type tends to be more complicated and also raises a number of difficult statistical questions. First, it is necessary to take into account the timing and order of events for the same subject. If one assumes that the first event is no different from the following events, then one may miss important and useful information regarding the timing and sequence of the repeated event (Box-Steffensmeier and Jones 2004). Second, the dependency between these recurrent events should be taken into consideration. As a subject experiences the same type of event more than once, the events from the same subject are potentially correlated (Amorim and Cai 2015; Wienke 2010). The occurrence of one event may change the probability of subsequent events of the same type. This means that the follow-up recurrent events are related to the occurrence of previous events.

The Cox model is not suitable for analyzing recurrent events because all events occurring after the first are neglected in the standard Cox model. It would be an inefficient use of data if we only make use of time to first event, ignoring subsequent events (Amorim and Cai 2015). In the analysis of recurrent data, all subjects are at risk of new events as long as they are not censored or have experienced a terminal event, while in the conventional survival analysis, individuals are not at risk after a first event (Commenges and Jacqmin-Gadda 2015).

There has been a recent surge of interest in modeling recurrent events in biomedical research. Marginal models and frailty models have been proposed by statisticians to account for recurrent events in survival analysis. Marginal models are appropriate when the substantive focus is on the effects of covariates, rather than the precise nature of the dependence structure as the association between events is considered as a nuisance parameter (Ezell et al. 2003; Liu, Wolfe and Huang 2004). However, in many real life circumstances, quantifying the dependence structure is the primary focus of research. Frailty models were developed to model the dependence structure of repeated events, and can also be easily incorporated into a joint modeling framework of multiple survival process, which will be discussed later in the paper.

2.3 Frailty Models

2.3.1 Frailty

Demographers James Vaupel and colleagues (1979) introduced the concept of frailty and applied it to the study of population mortality. They illustrated that the population hazard does not truly reflect the hazard of individuals from that population. They observed that at the oldest ages, mortality rates show a slower increase even though the hazards for individuals continue to increase. They explained this observed mortality rate decrease as a consequence of the failure-prone - more “frail”- individuals who die at younger ages leaving a subgroup of robust individuals at the older ages. Their work stimulated a growing body of research that gave priority to the concept of frailty with far-reaching implications (Myrskyl and Fenelon 2012; Palloni and Beltrn-Snchez 2017; Vaupel et al. 1998; Wrigley-Field 2014).

2.3.2 Univariate Frailty Models

In the context of survival analysis, frailty models are extensions of standard Cox model. To be more specific, frailty models are multiplicative hazard models consisting of three components: a frailty variable U (random effect), a baseline hazard function $h_0(t)$ (parametric or nonparametric), and a term modeling the influence of observed covariates $exp(\beta^T \mathbf{Z})$ (fixed effects) (Wienke 2010). The univariate frailty model is defined as follows:

$$h(t;\mathbf{Z}) = U h_0(t) exp(\beta^T \mathbf{Z}) \tag{2}$$

The frailty U is a nonnegative random variable varying over the population. Frailty distributions are standardized to $EU = 1$ (Wienke 2010). The variance $\sigma^2 = Var(U)$ is interpreted as a measure of heterogeneity across the population. When σ^2 is small, the values of U are closely located around one. When σ^2 is large, then values of U are more dispersed. Apart from the frailty variable U, all individuals are assumed to follow the same mortality pattern (Wienke 2010).

2.3.3 Shared Frailty Models

The role of frailty models in recurrent event analysis has received growing attention, where the correlation among event times is a focus of inquiry. The shared frailty model provides an efficient way to model this correlation by introducing a non-negative frailty variable, U, in the Cox model. The introduced frailty is considered to be shared among the events within the same subject to induce the dependence among them. All survival times that are related to each other have the same level of frailty. Conditional on the frailty, the event times within the same subject are assumed to be independent.

A shared frailty model in survival analysis is defined as follows. Suppose there are n independent individuals and that individual i has n_i observations and associates with the unobserved frailty u_i $(1 \leq i \leq n)$. The vector $\mathbf{Z}_{ij}$ $(1 \leq i \leq n, 1 \leq j \leq n_i)$ contains the covariate information of the event time T_{ij} of the jth observation for the ith individual. Conditional on the frailty term u_i, the survival times in individual i $(1 \leq i \leq n)$ are assumed to be independent and their hazard functions to be of the form:

$$r(t|u_i) = u_i r_0(t) exp(\beta^T \mathbf{Z}_{ij}) \tag{3}$$

The frailties u_i $(i = 1, ..., n)$ are assumed to be independently and identically distributed random variables following some distribution. Hougaard (2000) discussed the choice of frailty distribution for the shared frailty model, and noted that the gamma distribution has typically been used to fit the frailty random effect mainly due to mathematical reasons. That is, if one chooses a gamma distribution for the frailty, parameters estimates are easily obtained through likelihood estimation, which is also readily available in R packages.

2.3.4 Joint Modeling Framework

Different life events are potentially correlated via dynamic processes. People often take into account anticipated changes in one life event in making decisions in other life events. For example, union transition and childbearing within that union are two related dynamic processes. The decision to end a cohabitation or to move from cohabitation to marriage is likely to be jointly determined with the decision to have a child with that partner (Lillard and Waite 1993). Women might make greater investments in their relationship if they believe they will get married (Lillard and Waite 1993). Estimation of the impact of the anticipated events on current transitions is always challenging because these survival processes are endogenous. Observed and unobserved factors play roles in different event transitions. If decisions about these life events are jointly determined, then we might expect correlated unobserved factors of the models for each process. Joint modelling will serve the purpose as we can explicitly quantify the direction and magnitude of dependencies among these endogenous survival processes.

2.3.5 Joint Frailty Models

Joint analysis of recurrent event and survival time has received some research attention in recent years. Liu and colleagues (2004) proposed a joint shared frailty model and an estimation method based on the Monte Carlo EM algorithm. Along this line of inquiry, Rondeau et al. (2007) proposed jointly modeling the recurrent events and a terminal event using penalized likelihood estimation.

Following Rondeau et al. (2007), individual correlation between recurrent events and a terminal event is achieved by a shared frailty term. The model can be specified by the hazard functions:

$$\begin{cases} r_i(t|u_i) = u_i r_0(t) exp(\beta_1^T \mathbf{Z}_{i1}) & \text{(recurrent event)} \\ \lambda_i(t|u_i) = u_i^{\gamma} \lambda_0(t) exp(\beta_2^T \mathbf{Z}_{i2}) & \text{(terminal event)} \end{cases} \tag{4}$$

where $r_i(\cdot)$ denotes the hazard of the recurrent events and $\lambda_i(\cdot)$ denotes the hazard of the terminal event. A shared frailty term u_i links the two survival processes together. The random effects u_i are assumed independent and follow a gamma distribution with unit mean and variance θ.

The frailty term acts differently for the two hazard rates (u_i for the recurrent rate and u_i^{γ} for the terminal event rate). When $\gamma = 0$, the terminal event rate is independent of the recurrent event rate. When $\gamma = 1$, the effect of the frailty is the same for recurrent events and the terminal event.

3 THE MULTIVARIATE JOINT FRAILTY MODEL

As an extension to the joint frailty model for one type of recurrent event and a terminal event (Rondeau et al. 2007), Mazroui and colleagues (2013) introduce joint modeling of two types of recurrent events and a survival outcome. Individuals may experience multiple types of recurrent events in their lifetime. For example, marital dissolution can occur through either widowhood (death of a spouse) or divorce. Considering two types of repeatable events enables us to more accurately record a subject's marital transitions.

Following Mazroui et al. (2013), we present the setup and the estimation technique as follows. Let us consider two types of recurrent event times $X_{ij}^{(l)}$, $j = 1,...,n_i^{(l)}$ for subject $i = 1,...,N$; $l \in 1,2$ indicates the two types of recurrent events. Let C_i and D_i be censoring and death times. The number $n_i^{(l)}$ of observations for recurrent events of type l is a random variable. We denote each individual's terminal event time as T_i^* and $T_i^* = min(C_i, D_i)$ which could be a non-informative censoring C_i or death D_i. $T_{ij}^{(l)} = min(X_{ij}^{(l)}, C_i, D_i)$ corresponds to each follow-up time, $j = 1,...,n_i^{(l)}$. We consider event times as starting from age T_{i0} , which is assumed to be 0. $\delta_{ij}^{(l)} = I(T_{ij}^{(l)} = X_{ij}^{(l)})$ is a binary indicator for recurrent events, which is 0 if the observation is censored or if the subject had terminal event, and 1 if $X_{ij}^{(l)}$ is observed. Likewise, the death indicator is represented as $\delta_i^* = I(T_i^* = D_i)$. We observe $T_{ij}^{(l)}, T_i^*, \delta_{ij}^{(l)}, \delta_i^*$.

$N_i^{R(l)^*}(t)$ counts the number of recurrent events of type l for individual i over the interval $(0,t], i = 1,...,N$. Due to censoring, we cannot observe the true number of recurrent events experienced by the individual i. Instead, we observe the process $N_i^{R(l)}(t) = N_i^{R(l)^*}(min(T_i^*, t))$ which counts the observed numbers of recurrent events of type l. Similarly, we define the actual and the observed death indicators by time t as $N_i^{D^*}(t) = I(D_i \leq t)$ and $N_i^D(t) = I(T_i^* \leq t)$. Furthermore, let $Y_i(t) = I(t \leq T_i^*)$ denote whether or not the individual i is at risk of an event at time t. Over the small interval $[t, t+dt)$, the number of recurrent events that occur for subject i is $dN_i^{R(l)^*(t) = N_i^{R(l)^*}((t+dt)^-) - N_i^{R(l)^*}((t)^-)}$ and the number of observed recurrent events is $dN_i^{R(l)}(t) = Y_i(t)dN_i^{R(l)^*}$. Note that $n_i^{(l)} = N_i^{R(l)}(T_i^*)$.

The i^*th process up to time t is denoted by: $\mathscr{H}_{it} = \sigma\{Y_i(h), N_i^{R(l)}(h), l \in \{1,2\}, N_i^D(h), Z_i(h), 0 \leq h \leq t\}$, where $Z_i(h)$ is a vector of covariates. The intensity processes are jointly dependent through two correlated random effects u_i and v_i, which account for the unobserved heterogeneity, the inter-recurrence dependencies, and the dependency between different event types. We assume recurrent events and the terminal event cannot happen at the same time. In addition, we assume that the death precludes the observation of new recurrent events.

The recurrent event intensity processes at time t are, for $l \in (1,2)$: $Y_i(t)r_i^{(l)}(t)dt = P(dN_i^{R(l)}(t) = 1|\mathscr{H}_{it^-})$, where $r_i^{(l)}(t)dt = P(dN_i^{R(l)^*}(t) = 1|Z_i(t), u_i, v_i, D_i > t^-)$. The death intensity process at time t is: $Y_i(t)\lambda_i(t)dt = P(dN_i^D(t) = 1|\mathscr{H}_{it^-})$, where $\lambda_i(t)dt = P(dN_i^{D*} = 1|Z_i(t), u_i, v_i, D_i > t^-)$. Finally, we model the intensity functions of counting processes for the two types of recurrent events and the terminal event processes.

The multivariate frailty model for two types of recurrent events with a terminal event is given as follows:

$$\begin{cases} r_i^{(1)}(t|u_i, v_i) = r_0^{(1)}(t)exp(\beta_1' Z_{i1}(t) + u_i) & \text{(recurrence of type 1)} \\ r_i^{(2)}(t|u_i, v_i) = r_0^{(2)}(t)exp(\beta_2' Z_{i2}(t) + v_i) & \text{(recurrence of type 2)} \\ \lambda(t|u_i, v_i) = \lambda_0(t)exp(\beta_3' Z_{i3}(t) + \alpha_1 u_i + \alpha_2 v_i) & \text{(terminal event)} \end{cases} \tag{5}$$

where $r_0^{(l)}, l \in 1,2$, and $\lambda_0(t)$ are the recurrent and terminal event baseline hazard functions, and $\beta_1, \beta_2, \beta_3$ the regression coefficient vectors associated with three survival processes. Different covariates, whether time-varying or time invariant, could be incorporated into the hazards of the two recurrent events and the terminal event.

The multivariate frailty models are linked together by two correlated Gaussian random effects u_i, v_i:

$$\begin{pmatrix} u_i \\ v_i \end{pmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \theta & \rho\sqrt{\theta\eta} \\ \rho\sqrt{\theta\eta} & \eta \end{pmatrix}\right]$$

The random variable u_i is shared by the hazard of recurrent events of type 1 and the hazard of the terminal event. Therefore, the variance of u_i, θ, specifies the variability of the dependencies between occurrences of the recurrent events of type 1. Similarly, the random variable v_i is shared by the hazard of the recurrent events of type 2 and the hazard of the terminal event. The variance of v_i, η, specifies the variability of the dependencies between occurrences of the recurrent events of type 2. The sign and strength of the parameters α_1 and α_2 assess the relationships between two types of recurrent events and the terminal event respectively. The significance of this parameter estimate informs whether the two types of recurrent events and the terminal event are dependent.

Large magnitude of $\alpha_1(\alpha_2)$ illustrates strong dependency between recurrent events of type 1(2) and the terminal event. A high absolute value of the correlation coefficient ρ shows a strong dependency between the two types of recurrent events.

3.1 Estimation

In the frailty model, the parameter of interest (regression coefficients, the variance of random variables and the baseline hazard function) cannot be estimated by maximizing partial likelihood as in Cox Models because random variables are added to hazard functions. Several estimation approaches have been proposed: a frequentist approach using the EM algorithm (Klein 1992), a Bayeisan approach using Lapace integration (Ducrocq and Casella 1996) or the Markov Chain Monte Carlo algorithm (Clayton 1991). However, these methods are computationlly expensive with a large number of iterations. Rondeau et al. (2003) introduced a semi-parametric approach using penalized likelihood.

Unlike parametric models making strong assumptions about the shape of baseline hazard, the semi-parametric penalized likelihood methods are very robust. Rondeau et al. (2003) approximate baseline hazard functions by M-splines, and estimate parameters (β_1, β_2, $\beta_3, \theta, \eta, \alpha_1, \alpha_2$) and the baseline hazard functions $r_0^{(t)}, l \in 1,2$ for the recurrent events or $\lambda_0(t)$ for the terminal event by maximizing the penalized log-likelihood. They use minus the converged Hessian of the penalized log-likelihood to estimate the variance of the parameters. The details of likelihood construction are given in the Appendix A.

3.2 Goodness-of-fit

Martingale residuals have become popular in checking model adequacy in survival data (Commenges and Rondeau 2000). They enable us to check whether the model predicts accurately the number of observed events. For subject i and time t, they are defined as the differences between the number of events of subject i until t and the Breslow estimator of the cumulative hazard function of t (Rondeau, Mazroui and Gonzalez 2012). The residual can be interpreted as the observed number of events minus the expected number of events given the model at each t (Therneau and Grambsch 2000). The mean of the martingale residuals at a given time should be equal to zero.

3.3 Software

Our analyses are implemented in R, with the freely available package *frailtypack* (Krol et al. 2017). This package can be used to fit various joint models for survival events. In particular, *multivPenal*, a sub-routine in *frailtypack* fits models for two types of recurrent events and a terminal event. The estimates are obtained using a penalized log-likelihood approach, similar to the one developed by Rondeau et al. (2003). The package is available at the Comprehensive R Archive Network, http://cran.r-project.org/package=frailtypack.

4 AN APPLICATION TO MARITAL DISSOLUTION AND RETIREMENT

Most of the previous studies on retirement decisions have focused on the individuals, emphasizing economic factors (Becker 1991; Denaeghel et al. 2011; Johnson 2004). Pension eligibility and health limitations are the two main factors that have garnered the most attention (Szinovacz and Davey 2005; van den Berg et al. 2010). The economic literature tends to ignore the social context of retirement decisions. This is in contrast with much of the more recent sociological literature, which stresses the importance of incorporating contextual embeddings into the retirement decision-making process.

Sociologists consider retirement as a life-course transition that is both socially and temporally contingent (Moen et al. 2006; O'Rand et al. 2002; Pienta 1999; Szinovacz et al. 2001; Smith and Moen 1998). Life course theories emphasize that retirement experiences can be viewed as outcomes of an individual's earlier experiences in various life spheres, especially their occupational and family histories (Szinovacz and DeViney 2000). The focus is on the mutual influences of family members and the interdependence of life spheres in shaping outcomes across individual lives (Elder 1994; Moen et al. 1996; Raymo and Sweeney 2006). This interdependence is particularly meaningful in the study of women's lives, given the traditional role of women as care providers within the family. Women's lives are affected by decisions regarding family formation and dissolution (Moen et al. 1996; Raymo and Sweeney 2006).

To cast doubt on the efficacy of using a narrow economic focus to model the timing of retirement is to suggest examining family situations as well. A better understanding of differences in the retirement experience can be achieved if variations in the circumstances of family life and marital transitions are further explored.

Previous literature has long emphasized the union history and role of family background to the process of union dissolution (Amato 2010; Wu and Penning 2018). Conventionally, researchers either count the number of prior unions or pool intervals of unions together.

They fail to accurately record timing and sequencing of repeatable events. Whenever there are multiple observations from the same subject, these observations are likely to be statistical dependent. Pooling observations without taking the dependence into account can lead to loss of information (Allison 2010). We apply shared frailty models to correct for this dependence in this paper.

4.1 Data

We used data from the 2007 Canadian General Social Survey, Cycle 21 (GSS-21), conducted by Statistics Canada. Statistics Canada's GSS program is an annual national (cross-sectional) survey that gathers individual- and household-level data on Canadian adults to monitor changes in social conditions and the well-being of Canadians (Statistics Canada, 2009). The GSS-21 focuses on aging, family, and social support. It includes detailed data on retirement decisions, family history, childbearing history, social support, health conditions, and standard demographic and socioeconomic variables. Thus, it enables an in-depth exploration of how marital transition and instability are associated with retirement timing.

The target population included Canadians age 45 and older living in all 10 provinces, excluding those living in the northern territories and full-time residents of institutions. The data were collected through telephone interviews, using the random-digit-dialing (RDD) method.

The GSS-21 includes a nationally representative sample of 23,404 Canadians aged 45 and older, with an overall response rate of 57.7%. We restricted our study sample to female respondents who were in marriage at age 45. With this restriction, the sample size was reduced to 8491. Missing data (educational attainment) were imputed using the predictive mean matching method (Buuren and Groothuis-Oudshoorn 2010).

The GSS-21 collected detailed retrospective data on marriage formation and dissolution. For each marriage (up to four marriages), data were collected for when each began and how each ended (if it dissolved), and the dates of each marital status transition. Using these data, we were able to reconstruct a marital history for each respondent up to the time of the survey. Retrospective studies have some inherent limitations, for example they may not have data on all potential confounding factors. The events of interest had already occurred prior to the time of the survey and much of related information was collected only at the time of the survey. For example, socioeconomic variables are well-established factors of retirement decisions in later life (De Preter et al. 2013; Kubicek et al. 2010; O'Rand et al. 2002), but the data on socioeconomic resources were collected at the time of the survey, reflecting the respondent's current status, and do not necessarily reflect their socioeconomic resources at the time when they retired. In our study, efforts were made to include a proxy measure of economic well-being (pension), reducing potential bias.

4.2 Measurement

As we jointly model two recurrent event processes and one terminal event process, we have three dependent variables (one for each process). Exposure time to the risk of two repeatable events (widowhood and divorce) were measured from age 45 until the time of the survey. Up to two events of each types were recorded. Exposure time to the risk of the terminal event (retirement) was measured from age 45 until the age of retirement. All subjects are at risk of new recurrent events as long as they are not censored or experienced retirement. Once subjects experience retirement, they are no longer at risk.

Our analysis also considered several demographic and socioeconomic variables that are known risk factors of marital dissolution. Researchers have consistently observed that the presence of children is significantly related to risk of marital dissolution. However, the results are mixed. On the one hand, union stability might be strengthened if children provide emotional support and instrumental support to aging parents (Wu and Penning 2018). On the other hand, if older children have strained relationships with aging parents in terms of inheritance concerns and other issues, one might expect negative implications for union stability (Brown and Lin 2012). In addition, low socioeconomic status (education and income) and interracial couples are reported to have higher risk of marital dissolution (Atamo 2010; Brown and Lin 2012; Ross and Mirowsky 2010; Sweeney and Phillips 2004).

The descriptive statistics for the selected independent variables are presented in Table 1. The number of children ever raised by the respondent was included as a continuous variable. The mean number of children was 2.31.

We considered two socioeconomic variables. Educational attainment was an ordinal variable with ten levels, ranging from 1 (elementary school education or less) to 10 (some postgraduate education or higher). It was treated as a continuous variable in the analysis. The mean level of education was 5.35 (some post-secondary education) for women. Pension was a dummy variable indicating whether the respondent had a pension plan besides government-sponsored pensions (i.e., the Canada or Quebec Pension Plans). We observed that 40.68% of women had an employment-sponsored pension.

Racial minority status was also included as a dummy variable (1=visible minority), indicating whether the respondent is a racial minority (non-white). Table 1 shows that 6.22% of women in the data fell in this category.

Table 1: Descriptive Statistics

Variables	Mean or %	S.D.	Median
Number of Children	2.31	1.69	2.00
Education	5.35	3.00	5.00
Pension	40.68%	-	-
Racial Minority	6.22%	-	-
N	8491		

As noted, our analysis of the timing of retirement considers two types of recurrent events that may influence retirement decisions: widowhood and divorce. Table 2 shows the number of widowhood recurrences and divorce recurrences ranged from 0 to 2, given the short span of risk time. Table 2 also displays that 1245 women had widowhood events and 523 had divorces. The numbers of zero, one and two widowhood recurrences were 7246, 1223 and 22 respectively. The comparable figures of divorce recurrences were 8038, 442 and 11. For women, there were 2715 retirement events during the follow-up.

Table 2: Frequency of Widowhood and Divorce Recurrences

Number of Recurrences	0	1	2
Widowhood	7246	1223	22
Divorce	8038	442	11

4.3 Results

First, we ran three separate models for the three survival processes. We fitted two univariate frailty models for two types of recurrent events (widowhood and divorce) and a Cox model for terminal event (retirement). Second, we fitted a multivariate frailty model with baseline hazard functions approximated by M-splines to model jointly widowhood recurrences, divorce recurrences and retirement for women. Maximization of the penalized likelihood estimation method was used for the models for which the baseline hazard functions were approximated by M-splines. Finally, we investigated the dependencies between two types of recurrent events and the terminal event.

4.3.1 Separate Models

Considering the results presented in Table 3, the number of children is associated with woman's widowhood risk, controlling for our selected covariates. A higher level of education is associated with decreased risk of widowhood and increased risk of divorce. Both the risk of widowhood and the risk of divorce are not significantly different for women with pension versus not. A woman's visible minority status is associated with decreased risk of widowhood recurrences.

Table 3 also shows the estimated risk of retirement decreased for women who have more children. Women with pension plans have significantly higher estimated risks of retirement, with a hazard ratio of retirement of 2.36. Other socioeconomic factors, such as education, also show significant positive associations with retirement among women. Visible minority women have lower risks of retirement than non-visible minority women.

Table 3: Separate Models of the Widowhood and Divorce Recurrences and Retirement

Variables	Widowhood HR (95%CI)	Divorce HR (95%CI)	Retirement HR (95%CI)
Number of Children	1.09***	1.03	0.91***
	(1.06,1.12)	(0.98,1.09)	(0.88,0.93)
Education	0.95***	1.07***	1.05***
	(0.93,0.97)	(1.03,1.10)	(1.03,1.06)
Pension	1.04	1.16	2.36***
	(0.90,1.21)	(0.95,1.40)	(2.15,2.59)
Minority	0.61**	0.94	0.65***
	(0.43,0.86)	(0.63,1.38)	(0.52,0.81)
Penalized Marginal Loglikelihood	-6597.13	-3023.7	-11441.12

[a] Data are based on the 2007 Canadian General Social Survey

[b] $*p < 0.05, **p < 0.01, ***p < 0.001$ (two-tailed test).

4.3.2 A Joint Model

Table 4 presents the results for covariate estimates in the joint model. The estimated risk of widowhood increases for respondents with more children and decreases for respondents with a lower level of education and having visible minority status. Women with a higher level of education have greater risks of divorce recurrences. Women with more children and having visible minority status tend to retire at an earlier time. Also, the risks of retirement increase for women with higher education and those with pension plans.

The covariate estimates are very similar in the separate and joint models. The added value of the joint model is the frailty parameter estimates. The variance of divorce random effect (1.20) is much larger than the variance of widowhood random effect (0.07), which indicates the divorce responses are more heterogeneous. The sign and strength of the dependency between the recurrent event type 1(2) and the terminal event is represented by α_1 and α_2. The parameter estimates α_1 and α_2 are both significantly different from 0, meaning that there are positive and strong dependencies between the risk of widowhood (divorce) recurrences and the risk of retirement. Widowhood and divorce random effects lead to large hazard estimates for retirement. These frailties parameter estimates highlight the merits of our proposed model.

Table 4: A Joint Model of the Widowhood and Divorce Recurrences and Retirement

	HR	(95%CI)
Widowhood recurrences		
Number of Children	1.10***	(1.07,1.13)
Education	0.95***	(0.93,0.97)
Pension	1.04	(0.90,1.21)
Minority	0.62***	(0.43,0.87)
Divorce recurrences		
Number of Children	1.03	(0.97,1.08)
Education	1.06***	(1.03,1.10)
Pension	1.21	(0.99,1.48)
Minority	0.90	(0.60,1.36)
Retirement		
Number of Children	0.90***	(0.88,0.92)
Education	1.05***	(1.03,1.06)
Pension	2.45***	(2.25,2.66)
Minority	0.69***	(0.56,0.85)
Parameters associated with frailties		
Widowhood $\theta = var(u_i)(SE)$	0.07	(0.44)
Divorce $\eta = var(v_i)(SE)$	1.20	(1.10)
Widowhood $\alpha_1(SE)$	0.40***	(0.01)
Divorce $\alpha_2(SE)$	0.25***	(0.01)
Correlation (SE)	-0.95	(0.18)

[a] Data are based on the 2007 Canadian General Social Survey

[b] $*p < 0.05, **p < 0.01, ***p < 0.001$ (two-tailed test)

Moreover, we observe large negative correlation coefficients between two random effects at -0.95. Thus, we plotted estimated widowhood random effects against divorce random effects for women (see Figure 1). The Appendix B refers to the estimation of random effects. It is noteworthy that divorce random effects are larger than widowhood random effects. Therefore, large divorce rate hazards are related to small widowhood hazards, which results in large negative correlation coefficient.

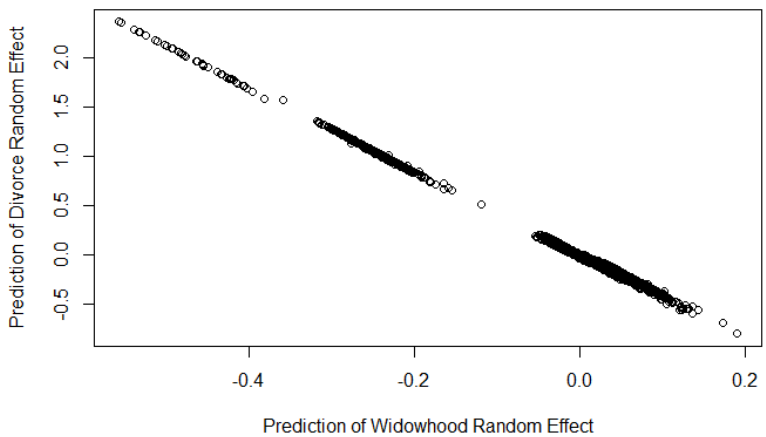

Figure 1: Scatterplot of random effects estimated for divorce versus widowhood

We also plotted the Martingale residuals for three survival processes against the follow-up time. Under the assumption of well fitted models, the Martingale residuals should have a mean equal to zero. The smoothing curve added to the plot should approximately overlap with the horizontal line $y = 0$. The results are shown in Figure 2, 3 and 4. We note that the means of the Martingale residuals for divorce recurrences are close to zero. The means of residuals for widowhood recurrences are a little below zero at the early follow-up times and are close to zero for the longer follow-up times. The tendencies of retirements are deviated by relatively lower values for longer follow-up times. This suggests that the models have overestimated the number of retirement in the longer follow-up period. Moreover, the smoothing of these residuals obtained with the Lowess function is close to the line $y = 0$ (Therneau and Grambsch 2000).

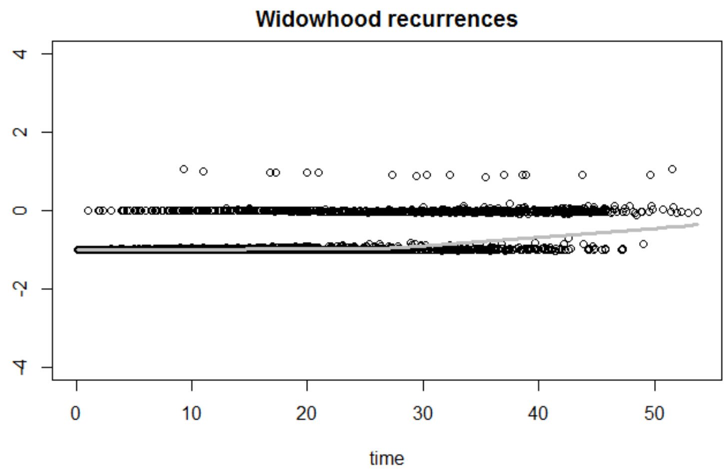

Figure 2: Martingale residuals for widowhood process against follow time

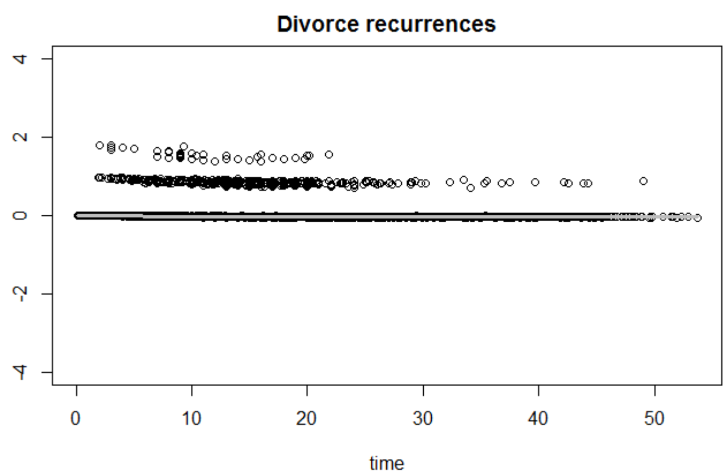

Figure 3: Martingale residuals for divorce process against follow time

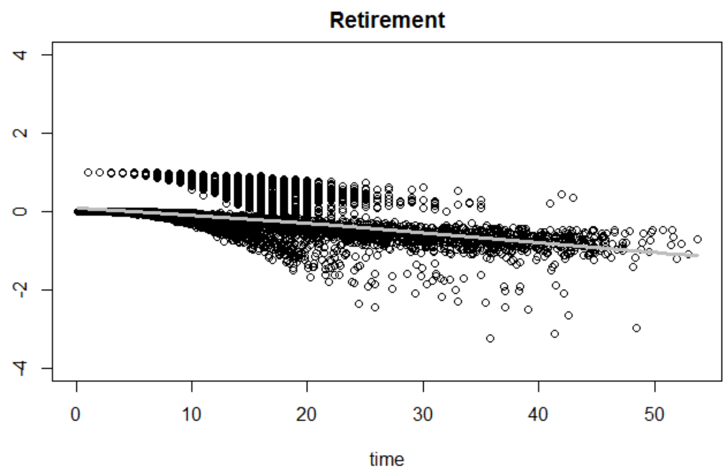

Figure 4: Martingale residuals for retirement process against follow time

5 Conclusions

In this study we introduced a multivariate frailty model with two correlated random effects to simultaneously model two types of recurrent events with a dependent terminal event. It has focused on the extent to which these processes are interrelated, given that they are subject to joint decision-making. Simultaneous modeling of multiple survival processes in joint models offered a number of advantages over separate modeling of each outcome. The model not only explicitly assessed possible dependencies among three survival processes but also corrected for biases introduced into regression estimates due to sources of endogeneity.

We applied this method to examine how the timing of retirement is associated with the risk of union dissolution. Rather than focusing on the outcome of first divorce or widowhood, our model considers all episodes of divorce or widowhood experienced by subjects from age 45 until the time of retirement (the terminal event) or the time of the survey if retirement had not occurred by the time of the survey. Second, we model transitions to widowhood and divorce and retirement jointly, thus allowing for the possibility that the risk of marital dissolution and the risk of retirement are both endogenous processes.

We believe this method will be particularly useful in two research scenarios. The first one is when the focus is on the survival outcome and we wish to account for the effects of recurrent events. For example, health benefits of marriage have long been recognized and extensively studied. However, previous research has yielded inconsistent results for older people (Brown and Wright 2017; Liu and Waite 2014). To our knowledge, very little research has considered effects of dynamic and complex marital history with possibilities of several entries to and exits from cohabitations and marriages. Other than counting the number of union transitions, our method can accurately record the timing and sequence of multiple cohabitations and marriages. It will be a powerful tool to answer research questions about how repeatable union transition affects health and longevity in the long run.

The second one is when multiple types of survival data are available. The joint model framework appropriately accounts for potential correlations among them. The method demonstrated in this paper provides a more nuanced understanding of correlated event processes, especially the issue of endogeneity in survival analysis. For example, Steele (2011) examined the relationship between employment transitions and births. She found that the number and age of children are associated with the timing of a non-employed woman's return to work. Along the same vein, promising research avenues would be exploring relationships between employment transitions and union formation or dissolution.

Future endeavors will be devoted to incorporating techniques for examining other frailty distributions. We have chosen a multivariate normal distribution for the random effects because it is flexible in modeling the covariance structure among different types of event. However, the impact of random effects misspecification in joint model framework warrants further investigation.

Appendix A Likelihood Construction

Following Mazoui et al. (2013), we first calculate the conditional contribution of the individual i to the likelihood.

$$L_i(\phi|u_i,v_i)=\prod_{j=1}^{n_i^{(1)}}P(X_{ij}^{(1)}=T_{ij}^{(1)}|X_{ij}^{(1)}>T_{i(j-1)}^{(1)},u_i,v_i)^{\delta_{ij}^{(1)}}P(X_{ij}^{(1)}>T_{ij}^{(1)}|X_{ij}^{(1)}>T_{i(j-1)}^{(1)},u_i,v_i)^{1-\delta_{ij}^{(1)}}$$

$$\times\prod_{j=1}^{n_i^{(2)}}P(X_{ij}^{(2)}=T_{ij}^{(2)}|X_{ij}^{(2)}>T_{i(j-1)}^{(2)},u_i,v_i)^{\delta_{ij}^{(2)}}P(X_{ij}^{(2)}>T_{ij}^{(2)}|X_{ij}^{(2)}>T_{i(j-1)}^{(2)},u_i,v_i)^{1-\delta_{ij}^{(2)}}$$

$$\times P(D_i=T_i^*|u_i,v_i)^{\delta_i^*}P(D_i>T_i^*|u_i,v_i)^{1-\delta_i^*}=\prod_{j=1}^{n_i^{(1)}}r_i^{(1)}(T_{ij}^{(1)}|u_i,v_i)^{\delta_{ij}^{(1)}}\frac{P(X_{ij}^{(1)}>T_{ij}^{(1)}|u_i,v_i)}{P(X_{ij}^{(1)}>T_{i(j-1)}^{(1)}|u_i,v_i)}$$

$$\times\prod_{j=1}^{n_i^{(1)}}r_i^{(1)}(T_{ij}^{(1)}|u_i,v_i)^{\delta_{ij}^{(2)}}\frac{P(X_{ij}^{(2)}>T_{ij}^{(2)}|u_i,v_i)}{P(X_{ij}^{(2)}>T_{i(j-1)}^{(2)}|u_i,v_i)}\times\lambda(T_i^*|u_i,v_i)^{\delta_i^*}P(D_i>T_i^*|u_i,v_i)$$

The random effects (u_i,v_i) are Gaussian and correlated with $var(u_i)=\theta, var(v_i)=\eta, corr(u_i,v_i)=\rho$, and $f(u_i,v_i)=\frac{1}{2\pi\sqrt{\theta\eta}\sqrt{(1-\rho^2)}}exp(\frac{-u_i^2/\theta+2\rho u_iv_i/\sqrt{\theta\eta}-v_i^2/\eta}{2(1-\rho^2)})$

We integrate the conditional contribution of the individual i:

$L_i(\phi)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}L_i(\Phi|u_i,v_i)\times f(u_i,v_i)du_idv_i$

Then we obtain the individual marginal contribution to the likelihood:

$$L_i(\Phi)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}exp\Big(-exp(u_i)\sum_{j=1}^{n_i^{(1)}+1}\int_{T_{i(j-1)}^{(1)}}^{T_{ij}^{(1)}}r_i^{(1)}(t)dt\Big)exp\Big(-exp(v_i)\sum_{j=1}^{n_i^{(1)}+1}\int_{T_{i(j-1)}^{(2)}}^{T_{ij}^{(2)}}r_i^{(2)}(t)dt\Big)$$

$$\times exp\Big(-exp(\alpha_1u_i+\alpha_2v_i)\int_0^{T_i^*}\lambda_i(t)dt\Big)\times(n_i^{(1)}u_i+n_i^{(2)}v_i+\delta_i^*(\alpha_1u_i+\alpha_2v_i))$$

$$exp\Big(\frac{-u_i^2/\theta+2\rho u_iv_i/\sqrt{\theta\eta}-v_i^2/\eta}{2(1-\rho^2)}\Big)du_idv_i$$

$$L_i(\Phi) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} exp\Big[-exp(u_i)\sum_{j=1}^{n_i^{(1)}+1}\int_0^{S_{ij}^{(1)}} r_i^{(1)}(t)dt - exp(v_i)$$

$$\sum_{j=1}^{n_i^{(1)}+1}\int_0^{S_{ij}^{(2)}} r_i^{(2)}(t)dt - exp(\alpha_1 u_i + \alpha_2 v_i)\int_0^{T_i^*}\lambda_i(t)dt\Big)$$

$$+\Big(\frac{-u_i^2/\theta + 2\rho u_i v_i/\sqrt{\theta\eta} - v_i^2/\eta}{2(1-\rho^2)} + (n_i^{(1)} + \delta_i^*\alpha_1)u_i + (n_i^{(2)} + \delta_i^*\alpha_2)v_i\Big)\Big]du_i dv_i$$

Appendix B Estimation of the Random Effects u_i, v_i

According to Mazoui et al. (2013), let $T_i = T_{ij}^{(l)}, j^{(l)} = 1, ..., n_i^{(l)} + 1$. The posterior probability density function is

$f(u_i, v_i|T_i, \hat{\phi}) = \frac{f(T_i|u_i, v_i, \hat{\phi}) * f(u_i, v_i|\hat{\phi})}{f(T_i|\hat{\phi})}$ and $f(u_i, v_i|T_i, \hat{\phi}) \propto f(T_i|u_i, v_i, \hat{\phi}) * f(u_i, v_i|\hat{\phi})$. Here $f(T_i|u_i, v_i, \hat{\phi})$ corresponds to likelihood of individual i given $\hat{\phi}$ and given the random effects u_i, v_i.

The mode of the posterior probability density function is obtained by maximizing it using Marquardt algorithm:

$$(u_i, v_i|T_i, \hat{\phi}) \propto exp\Big[\Big(-exp(u_i)\int_0^{T_i^*}\widehat{r_i^{(1)}}(u)du - exp(v_i)\int_0^{T_i^*}\widehat{r_i^{(2)}}(u)du$$

$$-exp(\widehat{\alpha}_1 u_i + \widehat{\alpha}_2 v_i)\int_0^{T_i^*}\widehat{\lambda}_i(u)du\Big)$$

$$+\Big(\frac{-u_i^2/\hat{\theta} + 2\hat{\rho}u_i v_i/\sqrt{\hat{\theta}\hat{\eta}} - v_i^2/\hat{\eta}}{2(1-\hat{\rho}^2)}(n_i^{(1)} + \delta_i^*\widehat{\alpha_1})u_i + (n_i^{(2)} + \delta_i^*\widehat{\alpha_2}v_i)\Big)\Big]$$

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