Replacing 2.1: Fertility Level and Long Run Population Growth in Countries with Net Immigration

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Abstract

Over 2010-15 the TFR was below 2.1 in all the More Developed Countries, except Israel. 65% of these countries also had positive net immigration. For 22 countries, this paper calculates a 'With Migration Replacement TFR' which equates the size of the stationary population to which convergence would occur over time under constant mortality and net migration amount to current population size, and a 'Replacement Migration' which does so under constant fertility and mortality. The results show the With Migration Replacement TFR ranges widely from 0.60 for Singapore to 2.05 for Slovakia, and is below the current TFR in 14 of the 22 countries. Despite its smaller population, absolute 'Replacement Migration' is higher for Japan than for USA, due to its lower TFR. On a per 1000 population basis 'Replacement Migration' is highest for Korea and lowest for France. Synthetic measures for the long run population growth implications of the combined effects of age profiles of fertility, migration and mortality for long run population growth are proposed and illustrated. Simple, short-cut approximate estimators of Migration Replacement TFR are proposed, with a view to their accessibility helping to counter a popular misconception that a TFR of 2.1 is necessary to prevent long-run population decline. The results demonstrate the importance of considering the implication of a particular level of national fertility in conjunction with the prevailing migration and mortality levels, as opposed to in isolation.

Introduction

For 2010-2015 all the More Developed Countries, except Israel, the Total Fertility Rate (TFR) was below 2.1, which is approximately the level which a country with constant low mortality, a typical sex ratio at birth, and zero migration, would need to maintain to prevent a long run population decline (Espenshade et al. 2004, UNPD 2017). In 65% of these countries net immigration was positive, with the exceptions mostly being Eastern European countries (UNPD 2017). The '2.1 rule' is inapplicable if positive net immigration is sustained over time. If a population which experiences constant fertility below exact replacement level[1], constant mortality and constant net immigration with a fixed age composition will, over the long run, converge to a stationary state with non-zero size, zero growth and constant numbers by age (henceforth referred to as the 'Terminal Stationary Population' (TSP)) (Cerone 1987; Espenshade et al 1982; Pollard 1973). However, an unqualified equating of a TFR below 2.1 with long run population decline appears to be common in popular and media discussion of fertility, even in countries in which positive net immigration has long been the norms[2].

The size of the TSP to which a population experiencing constant fertility below exact replacement level[3], constant mortality and constant (in absolute terms) net immigration for a specified population and time period may be seen as an intrinsic measure of the population size implication combination of fertility, mortality and net migration for that population and time (Espenshade et al. 1982). Espenshade (1982) notes on the possibility of calculating a migration number which equates TSP and current population size, and terms this number *Replacement Migration*[4]. *Replacement Migration* level may be calculated for any specified combination of constant age-specific fertility rates and constant age-specific mortality rates, provided the age-sex distribution of net migration is also specified.

This paper presents a parallel measure (to *Replacement Migration*) for fertility (henceforth the *With (positive net) Migration Replacement TFR* (TFR_R)), that is the constant TFR which in combination with constant mortality and constant net migration in absolute terms by age and sex at the estimated current levels equates a country's *TSP* size with its estimated current population size. The extension of the concept 'replacement' to consider constant fertility, mortality and migration *rates* (as opposed to absolute amounts) jointly has

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[1] Here constant fertility includes constant sex ratio at birth.

[2] See for example Fox (2018), NSTP (2018), Gallagher (2018), Smith (2019)

[3] Here constant fertility includes constant sex ratio at birth.

[4] The term Replacement Migration has also been attached to other measures in the literature. UNDP (2001) use the term variously and differently, including for the migration required to maintain the size of the total population at the highest level it would reach in the absence of migration after the initial year of their population projection. Billari and Dalla-Zuanna (2011) use it in relation to maintaining the size of birth cohort.

been considered by Preston and Wang (2007), who extended the conventional (zero migration) Net Reproduction Rate (henceforth NRR) to incorporate age-specific rates of net migration, and calculated an associated intrinsic growth rate. However, the use of age-specific rates of net migration as data inputs to population projections for countries in which net migration is anticipated to be positive appears to be considerably less common than the formulation of such assumptions in terms of absolute numbers (UNPD 2017). Despite its simplicity of calculation using readily-available data, the use of the Preston and Wang's (2007) NRR* to date appears to have been limited. Various indicators of birth cohort intergenerational (mothers and daughters) replacement have also been proposed in the literature (Billari and Dalla-Zuanna 2011; Del Ray Poveda and Cebran-Villar 2002; Wilson et al. 2013).

The paper, for the first time, presents[5] and analyses the variation in the values of the *With Migration Replacement TFR* (TFR_R), using recent data for an extensive range of More Developed Countries[6]. It also presents and analyses the variation in *Replacement Migration* (M_R) for these countries. In view of a possibility that the complexity of the formulation of stationary populations with immigration may have been detrimental the use of such models to counter the seemingly persistent the popular misconception that a TFR of 2.1 is necessary to prevent long-run population decline, irrespective of immigration level, in addition to specifying the 'exact' formula, this paper evaluates two relatively simple, rule-of-thumb formulae for calculating *With Migration Replacement TFR* which may be calculated quickly (e.g. using a smart 'phone), using only the more widely-known and more readily accessible demographic measures as input variables.

Method

The TSP size (denoted P_A) which corresponds to sustained constant (absolute) net immigration by age and sex, constant age-sex specific mortality rates, constant below-replacement fertility with a constant proportionate age distribution and a constant sex ratio at birth at the levels observed for a particular population and time period A can be expressed as the sum of components corresponding to generations of migrants (Schmertmann 1992). A person's migrant generation index is based on the most recent foreign-born individual to migrate into population A out of the set comprising the person plus his/her all female line of

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[5] To the authors' knowledge, the formula for calculating this remains undocumented.

[6] To the authors' knowledge the cross-country comparison of Replacement Migration in this paper is also unique.

ancestry. A population can be partitioned into migrant generations. Thus P_A equals the sum of migrant generation sizes:

$$P_A = \sum_{i=1}^{\infty} P_{i,A} \tag{1}$$

Where P_A denotes the total size of the stationary population, and i is the migrant generation index, and $P_{i,A}$ the size of the ith migrant generation. Thus $P_{1,A}$ denotes the stock of (first generation) immigrants, $P_{2,A}$ the stock of native-born of immigrant mother, $P_{3,A}$ the stock of native-born of native-born mother with immigrant mother, and so on.

For constant net migration with constant, non-zero amounts of emigration[7] parallel components of population size can be calculated. However literal correspondence between 'migrant generation' components and sets of people categorised by ancestry no longer applies. The calculation of the various generation sizes, and hence TSP size, in this paper uses discrete approximations to formulae in Schmertmann (1992) which are readily calculated from widely-available national and international statistical agency data. The 'first generation' element in Eq. (1) ($P_{1,A}$) is calculated as:

$$P_{1,A} = M_A \sum_{j=1}^{2} \sum_{x=0}^{\omega} m_{x,j,A}\, e_{x,j,A} \tag{2}$$

Where M_A denotes the constant annual total net migration for A, $m_{x,j,A}$ denotes the proportion of total net migration contributed by persons of age x (last birthday) and sex j (j = 1 denotes female and j =2 male) for A, $e_{x,j,A}$ is the (remaining) life expectancy for age x and sex j for A[8], and ω denotes the maximum age for that population.

The 'second generation' element in Eq. (1) ($P_{2,A}$) is calculated by:

$$P_{2,A} = M_A\, TFR_A \sum_{j=1}^{2} s_{j,A}\, e_{0,j,A} \sum_{x=0}^{k} m_{x,1,A} \sum_{t=0}^{k-x} f_{x+t,A}\; {}_{t}p_{x,1,A} \tag{3}$$

Where TFR_A denotes the Total Fertility Rate (per woman) for A, $f_{x+t,A}$ represents the proportionate contribution to TFR_A from the age-specific fertility for age $x+t$, ${}_{t}p_{x,1,A}$ denotes the probability of a female surviving from x to $x+t$, k denotes the upper limit of the female reproductive age range, $s_{j,A}$ denotes the proportion of births of sex j, and $e_{0,j,A}$ denotes life expectancy at birth for sex j.

Thus the annual births in TSP_A (denoted B_A) is calculated by:

$$B_A = M_A\, TFR_A \sum_{x=0}^{k} m_{x,1,A} \sum_{t=0}^{k-x} f_{x+t,A}\; {}_{t}p_{x,1,A} \tag{4}$$

For all i ≥ 2

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[7] There are various alternative possibilities for the formulation of net migration, including as a rate or with immigration formulated in terms of an amount combined with emigration formulated as a rate (Preston and Wang 2007, Ryder 1997). For some countries considered here, a lack of data on emigration precludes the use of separate consideration of immigration and emigration.

[8] For simplicity differences in mortality (and fertility) by migrant generation are deliberately ignored.

$$P_{i+1,A} = NRR_A\, P_{i,A} \tag{5}$$

where NRR_A denotes the conventional (with zero migration) NRR for A. The sum of the sizes of the generation-indexed components for generations with indices 2 and above is the sum of a geometric series with initial term $P_{2,A}$ and common ratio NRR_A. Hence substituting from Eq. (5), Eq. (1) can be re-expressed as:

$$P_A = P_{1,A} + \frac{P_{2,A}}{(1-NRR_A)} \tag{6}$$

The TFR (denoted $TFR_{R,A}$ and henceforth termed the *With (Current) Migration Replacement* $TFR_{R.A}$[9]) which in combination with the values of M_A, $m_{x,j,A}$, $e_{x,j,A}$, $s_{j,A}$, $f_{x+t,A}$ and ${}_{t}p_{x,1,A}$ used in Eq. (1)-(4) equates the TSP size to the current population size POP_A can be calculated using the following equation:

$$TFR_{R,A} = \frac{TFR_A(POP_A - P_{1,A})}{\big(NRR_A(POP_A - P_{1,A})\big) + P_{2,A}} \qquad = \frac{TFR_A}{NRR_A} \times \frac{POP_A - P_{1,A}}{POP_A - P_{1,A} + \frac{P_{2,A}}{NRR_A}} \tag{7}$$

The derivation of Eq. (7) is presented in Appendix A. From Eq. (7) $TFR_{R.A}$ may be seen as the (conventional with zero migration) exact replacement level for A ($\frac{TFR_A}{NRR_A}$) multiplied by a value which depends on the combined values of the first ($P_{1,A}$) and second ($P_{2,A}$) generation components of the TSP for A, and its NRR. $TFR_{R.A}$ will be strictly less than $\frac{TFR_A}{NRR_A}$ when $P_{2,A}$ is positive. Technically, for this to occur the cumulative numbers for surviving female net migration to the reproductive ages must be positive[10]. However, since for all observed values of proportionate age-sex profile of net migration ($m_{x,j,A}$) in this paper, the value of $P_{2,A}$ is positive for any positive total net migration (M_A), it appears that in practice total net migration being positive is a sufficient condition for $TFR_{R.A}$ to be strictly less than $\frac{TFR_A}{NRR_A}$.

From Eq. (7), as net migration at all ages approaches zero (and hence the values of $P_{1,A}$ and $P_{2,A}$ approach zero) the value of $TFR_{R,A}$ approaches the value of $\frac{TFR_A}{NRR_A}$.

$TFR_{R,A} = 0$ if and only if $POP_A = P_{1,A} = M_A \sum_{j=1}^{2} \sum_{x=0}^{\omega} m_{x,j,A} e_{x,j,A}$[11]. (8)

From Eq. (8) the constant level of net migration which equates the *TSP size* with the current population size (POP_A) under constant proportions of net migration by age and sex $m_{x,j,A}$, constant mortality $e_{x,j,A}$ and $TFR_A = 0$ is calculated by:

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[9] In this paper only the TFR which equates TSP size with a recent estimate ('current') is considered. The more general term *Target Migration Replacement TFR* is suggested for use in examples in which either the population size or the net migration level is not based on current levels.

[10] Under weighting based on age-specific fertility rates.

[11] Discrete formulation, as opposed to continuous formulation, has been adopted throughout this paper, because of its greater compatibility with calculation from tabular data, and ease of comprehension by a wider readership.

$$M_{0.A} = \frac{POP_A}{\sum_{j=1}^{2} \sum_{x=0}^{\omega} m_{x,j,A}\, e_{x,j,A}} \tag{9}$$

Since, for any $M_A > M_{0,A}$. $P_A > POP_A$, and the *With Migration Replacement TFR* is not defined, the term *Fertility Superfluous (for growth) Level* is proposed for the net migration given by $M_{0,A}$. The *Index of Net Migration to Fertility Superfluous Level* (I_{NMFS}) = $\frac{M_A}{M_{0,A}}$.

Using a term of Ryder (1997), *Life Expectancy after Net Migration* (eNM_A) for persons for population A is defined:

$$eNM_A = \sum_{j=1}^{2} \sum_{x=0}^{\infty} m_{x,j,A} e_{x,j,A} \; = \frac{P_{1,A}}{M_A} = \frac{POP_A}{M_{0,A}} \tag{10}$$

The size of the first generation component of a TSP $P_{1,A}$ is the product of the total net migration (M_A) and *Life Expectancy after Net Migration* (eNM_A).

Life Expectancy after Net Migration ($eNM_{j,A}$) is also defined for each sex separately. eNM_A equals the average of the female ($eNM_{1,A}$) and male ($eNM_{2,A}$) values of life expectancy from age of net migration, weighted by the proportions *of net migration* by sex.

$$eNM_A \; = \frac{(M_{A,1} eNM_{A,1} + M_{A,2} eNM_{a,2})}{M_A} \tag{11}$$

where $M_{A,j}$ denotes the total net migration for sex j, and $eNM_{j,A}$ denotes L*ife Expectancy after Net Migration* for sex j.

The *Index of Life Expectancy after Net Migration* ($I_{ENM,j,A}$) for sex j is defined[12]:

$$I_{ENM\,j,A} = \frac{eNM_{j,A}}{e_{0,j,A}} \tag{12}$$

Thus *Life expectancy after Net Migration* ($eNM_{j,A}$) for sex j is the product of life expectancy at birth ($e_{0,j}$),,and an indicator of the implication of the combination of the distribution of ages of life table deaths and the *proportionate* distributions of net migration by age for years of life post (net) migration for j ($I_{ENM,j,A}$).

The denominator to be used in calculating $I_{ENM,A}$ on a per person basis is the average of male and female life expectancies at birth weighted by the proportions of *net migration* by sex.

A measure for fertility *Total Fertility Rate after Net Migration* (denoted $TFRNM_A$) which parallels eNM_A is:

$$TFRNM_A = (TFR_A \; \sum_{x=0}^{k}(m_{x,1,A} \sum_{t=0}^{k-x} f_{x+t,A}) \;)/(\; \sum_{x=0}^{\Omega} m_{x,1,A})$$

$$= (TFR_A \; \sum_{y=0}^{k}(f_{y,A} \sum_{x=0}^{y} m_{x,1,A}) \;)/(\; \sum_{x=0}^{\Omega} m_{x,1,A}) \tag{13}$$

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[12] Values of $I_{ENM,j,A}$ can exceed 1. For example, when net migration is positive at most younger ages (x) (which typically have higher values of $e_{x,j,A}$) and negative at older ages the values of $m_{x,j,A}$ can exceed 1 for the former and will be negative for the latter.

The value of *TFRNM* will be greater for younger age profiles of female migrants and greater for older age profiles of the TFR[13].

The *Index of TFR after Net Migration* ($I_{TFRNM,A}$) is the ratio of fertility from the ages of net migration to the TFR

$$I_{TFRNM,A} = \frac{TFRNM_A}{TFR_A} \tag{14}$$

The value of $I_{TFRNM,A}$ indicates the effect of the *proportionate* distributions of female net migration and the TFR by age on the average number of births of a (synthetic) age cohort of female migrants. It captures an aspect of the implication of the combined fertility and migration patterns for population growth which is separate from the scale of either the TFR or the total for net migration.

From Eq. (4) a similar (TSP) Index of Births per unit of net migration and unit of TFR ($I_{B,A}$) is:

$$I_{B,A} = \frac{B_A}{M_A TFR_A} = \sum_{x=0}^{k} m_{x,1,A} \sum_{t=0}^{k-x} f_{x+t,A}\ {}_t\mathrm{p}_{x,1,A} \tag{15}$$

Thus unlike $I_{TFRNM,A}$, the variation between countries in the Index of Births ($I_{B,A}$) will be affected by the sex ratio of net migration and the (generally very small) variation in probabilities of survival from age at migration to age at birth.

Eq. (3) can be expressed more concisely as:

$$P_{2,A} = I_{B,A}\, M_A\, TFR_A \sum_{j=1}^{2} s_{j,A} e_{0,j,A} \tag{16}$$

Substituting from Eq. (10)-(12) and (15) into Eq. (7) gives the following expression for $TFR_{R,A}$:

$$TFR_{R,A} = \frac{TFR_A}{NRR_A} \times \frac{POP_A - (M_A \sum_{j=1}^{2} \alpha_{j,A} I_{eNMj,A} e_{0,j,A})}{POP_A - (M_A \sum_{j=1}^{2} \alpha_{j,A} I_{eNMj,A} e_{0,j,A}) + (M_A \frac{TFR_A}{NRR_A} I_{B,A} \sum_{j=1}^{2} s_{j,A} e_{0,j,A})} \tag{17}$$

Where $\alpha_{j,A} = \frac{M_{j,A}}{M_A}$ represents net migration of sex *j* as a proportion of total net migration. Eq. (10), (12), (15) show that long run population growth is not only a matter of the magnitudes of TFR, life expectancy at birth and total net migration but is also affected by combinations of their age-sex distributions, for which $I_{eNM,A}$ and $I_{B,A}$. provide synthetic measures.

Since, from Eq. (2) and (3), M_A is a scalar value used in all the calculation of all the generation-indexed components ($P_{i,A}$) of TSP size P_A in Eq. (1), the *Replacement Migration* level ($M_{R,A}$) which in combination with the specified values for $m_{x,j,A}$, $e_{x,j,A}$, TFR_A, $s_{j,A}$, $f_{x+t,A}$

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[13] Unlike the conventional TFR, $TFRNM_A$ is affected by cohort numbers above age 50. It is possible for the value of $TFRNM_A$ to exceed the value of TFR_A and thus for $I_{TFRNM,A}$ to exceed 1. In particular, if cumulative female net migration above a specified female reproductive age x is negative in equation (13) then $\sum_{x=0}^{y} m_{x,1,A} >$ ($\sum_{x=0}^{\Omega} m_{x,1,A}$) and age-specific fertility rates for ages x and above are multiplied by a value greater than 1. '

and ${}_{t}p_{x,A}$ for A equates *TSP* size with the (mid-period) actual population size for the time period for which these fertility, mortality and net migration is observed (denoted POP_A) is simply expressed by:

$$M_{R,A} = \frac{M_A POP_A}{P_A} \tag{18}$$

This paper compares across countries the following: (i) *Terminal Stationary Population size* (*TSP*), (ii) *With Migration Replacement TFR* (TFR_R), (iii) *Fertility Superfluous Level* for net migration (M_0), and (iv) *Replacement Migration Level* (M_R). For all countries data for the same recent time period are used in the calculations. The analysis uses the variation in the and *Net Migration TFR Index* (I_{NOMTFR}), *Index of Births* ($I_{B,A}$) and *Life expectancy after Net Migration* ($eNM_{A,j}$), as well as the variation in the more familiar TFR, life expectancy at birth, and total net migration to account for the differences between countries.

In addition, two simple, short-cut methods for estimating the value of TFR_R are evaluated:

1) a simple regression model of TFR_R regressed on total net migration divided by current population.
2) combining the actual values of life expectancy at birth for each sex and total net migration for a particular country with the average values across all countries of the other elements in Eq. (17).

Both (1) and (2) use only widely-known and easily-accessed variables as data inputs. Neither method requires extensive, time-consuming data entry. Moreover, both methods allow an estimate of TFR_R, to be calculated quickly by basic arithmetic using a pocket calculator (or a smartphone). The accuracy of each method is assessed using the Mean Absolute Error (MAE) and the Mean Absolute Percentage Error (MAPE).

Excel spreadsheets which calculate the values of the various measures defined in this section from standard life table, age-specific fertility and net migration by age and sex input data will be made freely available via the internet.

Data

22 countries are considered here: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Hong Kong, Hungary, Italy, Japan, Korea (Republic of), Netherlands, New Zealand, Norway, Singapore, Slovakia, Sweden, Switzerland, United Kingdom (UK), and United States (USA). All the countries included are More Developed (according to the United Nations) with a 2013 total population size exceeding 1 million, and had both below replacement fertility and positive net migration based on the mean fertility and migration

levels for the 2011-2015 period. The mortality rates and population size were for time periods centred on 2013. The data were sourced from the websites of Eurostat, official national statistical offices, the OECD, the United Nations and the World Bank.

The TFR range from 1.23 for Singapore and 1.24 for Korea to 2.02 for New Zealand (Table 1). Along with France, the English-speaking countries (except Canada) and the Scandinavian countries generally have the relatively high TFRs, whilst the East and South East Asian, Eastern European and Central European countries (and Italy) the lower TFRs (Rindfuss et al. 2016). Mean age at birth is lowest for USA (28.7) and is also relatively low for Hungary and Slovakia. It is highest for Switzerland (31.8) and also relatively old for the four Asian populations considered and for Italy.

In absolute terms annual net migration ranges from 1.5 thousand for Slovakia to 959.8 thousand for the USA (Table 1). On a rate per 1000 population basis, net migration is highest for Singapore (12.5), Norway (9.5), Australia (8.7) and Canada (7.7), and lowest for Slovakia (0.3), Japan (0.6), France (0.9) and Hungary (1.1). For half the 22 countries females outnumber males in net migration (Table 1). France, Hong Kong, and Netherlands have the lowest sex ratios for net migration and Korea, Slovakia and Germany the highest. For both sexes life expectancy at birth is highest for Hong Kong, and significantly lower for Hungary and Slovakia than for the other countries considered (Table 1). For four of the 22 countries the proportionate age-sex distribution of net international migration was imputed from other sources, due to a lack of publicly-available with the necessary age detail from official sources[14].

Results

Terminal Stationary Population Size

For the countries with TFRs below 2.1 and which also have net immigration considered, it is more often the case that, with continued fertility at the 2011-2015 average level and also continued net immigration and mortality levels, it is a long run population increase, and not as is often assumed, a population decrease which is in prospect. Table 2 presents the *TSP size*

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[14] For France the proportions of net international migration by age for each sex were imputed from population estimates using a life table survival method, and then multiplied by estimates of total male net migration and total female net migration which were available from Eurostat. (Edmonston and Michalowski 2004). For USA estimates of net migration were derived using a life table survival method. For Hong Kong the proportionate age-sex distribution of inmovers with one-way permits from official data was used. For Singapore the average proportions of net migration by age for the 14 countries for which such data is readily available were applied to estimates of total net international migration which was calculated from published data on population change minus natural increase.

and its ratio to the 2013 (henceforth ‘current’ population size) for the countries considered. In absolute terms the TSP size is largest for the United States (430.9 million), followed by the United Kingdom (153.3 million), Australia (134.5 million) and France (112.4 million), and smallest for Slovakia (0.2 million), Hungary (1.4 million) and Hong Kong (2.8 million).

For 14 of the 22 countries considered the *TSP size* exceeds the current population size. The ratio of the *TSP* to current population size varies widely between countries. This ratio is highest for New Zealand (the TSP is 9 times the current population) Moreover, *TSP size* is more than double the current population for Australia, Norway, Sweden and UK. In contrast, for Slovakia, Japan, Korea and Hungary the TSP size is less than a seventh of the current population size. These variations show the long run population growth implications of below (zero migration) replacement fertility varies widely from population-to-population when considered jointly with the current migration and mortality levels.

Of the variables in Table 1 the one with the highest correlation with the ratio of TSP to current population is the TFR, followed by the net migration rate (Tables 1 and 2). Despite six other countries having a higher rate of net migration, New Zealand has the highest ratio of TSP to current population size, due to its having a higher (and near zero migration replacement) TFR[15]. Conversely, low fertility is a major contributor to Slovakia, Hungary, Japan. Korea and Italy having low TSP to current population size ratios.

Higher rates of net migration are also associated with higher TSP to current population size ratios (Tables 1 and 2). Net migration totals which are high in proportion to current population sizes contribute the high TSP to current population size ratios for Australia and Norway, whilst very net low migration relative to current population contributes to the low ratios of TSP to current population size for Japan, Korea, Slovakia and Hungary. The effect of the number of female migrants on TSP size is far greater than the effect of male migration[16] (which only affects the size of the first generation component ($P_{1,}$ in Eq.(1)). Relatively low life expectancies at birth also contribute to the low ratios of TSP to current population size for the latter two countries.

As well as TFR, net migration and life expectancy at birth, the variation in 2nd and higher order generation sizes between countries is also affected by the variation in the Index

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[15] Even small reductions in TFR (and NRR) below the level for New Zealand substantially increase the denominator of the (1/(1-*NRR)*) term in Eq. (6) and hence substantially reduce the combined size of generations 2 and above.

[16] From Eq. (1)-(6), the total female net migration affects the size of all migrant generation components, whereas the total male net migration affects only the first generation component (P_1).

of Births Index (I_B)[17] (Table 3). The value of I_B is lowest for Korea, due to its having both a relatively small proportion of females in its net migration and a relatively small proportion of its TFR which is accumulated post (net) migration, indicated by the value of the Index of TFR after Net Migration (I_{TFRNM}). Its relatively low value for I_{TFRNM} is due to the relatively old age profile of female migrants, and is despite its relatively old profile of age-specific fertility (Tables 1 and 3). The values of I_B, I_{TFRNM} and I_{eNM} all are highest for Netherlands and Belgium, both of which have young age profiles for female net migration. Low sex ratios of net migration also contribute to their high values of I_B and I_{eNM}. The effects on TSP size of the differences between the age-sex profiles of net migration of Netherlands and Belgium and those of other populations are substantial (Arthur and Espenshade 1988; Schmertmann 2012). If, their distinctive proportionate age-sex distributions for net migration are replaced by an average age-sex distribution based on the other countries in calculation the TSP size for Netherlands is just 62% as large (8.9 million compared to 14.3 million) and Belgium's is 75% as large (14.4 million compared to 19.1 million).

Current Migration Replacement TFR

Of the 22 countries, TFR_R is least for Singapore (0.60 births per woman), Norway (0.96) and Australia (1.00) (Table 2). These countries also have the highest net migration rates (Table 1). For these countries even reduction in TFR to 'lowest low' levels would result in a long-run population size below the current size, provided levels of (young and predominantly female) net migration and life expectancies at birth remain at least at current levels (Billari and Kohler 2004). TFR_R is highest, and near to the (zero migration) replacement level, for Slovakia (2.05), Japan (2.02) and Hungary (1.99) and Korea (1.98). Along with France, these countries have the lowest net migration rates (Table 1). However, whereas the net migration for France is predominantly female, the net migration to Japan, Slovakia, Hungary and Korea is predominantly male, which contributes to fewer births (Table 1). For eight countries the TFR_R falls in the 'very low fertility' range (i.e. below 1.5) (Billari and Kohler 2004).

The TFR_R is below the current TFR for 14 countries, and above it in the remaining eight (Fig. 1). Thus most of the countries considered have no need to increase their TFR at all if order to prevent a smaller than current long run population, provided the current levels of net migration and life expectancy are at least maintained. All the English-speaking countries and the countries of north, west and central Europe, except Finland and Netherlands have a

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[17] See Eq. (15).

TFR above TFR_R and all the countries of south and east Europe and Asia, except for Singapore a TFR below TFR_R. In absolute terms, the current TFR exceeds TFR_R by the widest margin for Australia (current TFR is 0.89 above TFR_R), Norway (0.84), and Singapore (0.63), and is furthest below TFR_R for Korea (current TFR is 0.74 below TFR_R), Slovakia (0.67), Hungary (0.63), and Japan (0.61) (Tables 1 and 2). The ratio of current TFR to TFR_R is highest for Singapore (current TFR is 2.05 times TFR_R), Australia and Norway (both 1.88 times), and least for Korea (0.63). For France even though the current TFR is only 0.05 (3%) above the TFR_R, the TSP exceeds the current population by 71%. This is largely due to the sensitivity of the combined 2nd and higher order generation component of TSP size (P_{2+} in Eq. (7)) to small changes to fertility which is near the (zero migration) replacement level[18]. Fig. 2 illustrates the strong negative correlation between TFR_R and net international migration rate per 1000 current population.

Fertility Superfluous Level

The current net migration is below the *Fertility Superfluous* level for every country considered. The *Index of Net* M*igration to Fertility Superfluous Level* (I_{NMFS}) ranges between 0.01 for Slovakia and 0.05 for Hungary to 0.51 for Australia, 0.54 for Norway and 0.72 for Singapore (Table 2). Thus, for example, even with a TFR of zero the migration level and mortality patterns Singapore which is 72% of the current population size. That 72% of the current population size would be 'replaced' by the first generation component (P_1) helps to explain why the TFR_R for Singapore (0.60) is so low (Table 2). The variation in this ratio mainly reflects the variation in net migration rates between countries (Table 1).

When expressed per 1000 current population the *Fertility Superfluous* levels are quite similar for most of the countries considered, with the value lying between 17.0 and 19.5 for 16 countries. Netherlands (14.7 per 1000 population), Belgium (15.7) and France (16.2) have the lowest *Fertility Superfluous* levels, due to their younger age profiles of net migration, and Hungary has the highest fertility superfluous migration rate (21.1), due to a combination of relatively old net migration and relatively high mortality (Tables 1 and 2). According to United Nations data, the net migration rate over 2010-15 five countries (Qatar, Lebanon, Kuwait, Jordan and Oman) exceeded even the highest *Fertility Superfluous* net migration

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[18] That the effect of any specified same absolute difference in TFR in Eq. (7) is larger when the values of the two TFRs are higher (and nearer to conventional (zero migration) replacement level) is mostly due to the change in the value of the (1/(1-NRR)) term.

level per 1000 population across the 22 countries studied (i.e. for Hungary), and that for Luxembourg also exceeded the mean (18.1) for the 22 countries (UNDP 2017).

Replacement Migration Level

Even though the current population of USA is two-and-a-half times that of Japan, in absolute terms Japan has the higher *Replacement Migration* level (M_R), mainly because of its lower TFR (Tables 1 and 2). Despite its being the third highest in absolute terms, M_R for Germany is slightly below the average net migration over 2011-2015, which was increased by a large inflow of refugees in 2014. The value of M_R is least for New Zealand and Norway, which both have small populations and high TFRs (Tables 1 and 2). The ratios of current migration are the same as those for TSP size to current population size, described earlier[19].

Replacement Migration per 1000 population is highest for Korea (9.8 per 1000 population), Hong Kong (8.4), Hungary (8.2) and Singapore (7.2) and lowest for France (0.5), New Zealand (0.7) and Australia (1.0). Thus across all 22 countries even the highest rate of value of M_R per 1000 population is significantly below the net migration rate which Singapore actually experienced over 2011-2015. The value of M_R *per 1000 population* is inversely related to the current TFR (Fig. 3; Table 2). However some variation in M_R *per 1000 population* between populations with the same TFR are evident. For example, despite both Denmark and Netherlands having a TFR of 1.7, M_R is 3.3 for Denmark and 2.0 for Netherlands. The lower M_R for Netherlands is linked to its younger and more feminine net migration, and hence higher values of I_B and I_{ENOM}. Despite its lower TFR, M_R per *1000 population* is also lower for France than for New Zealand, because France has predominantly female net migration, whereas New Zealand's net migration is predominantly male.

Whether a country's net migration is above or below replacement is associated with its TFR. All eight countries with a TFR above 1.76 are have above replacement net migration, whilst of the seven only countries with a TFR below 1.41 only Singapore is above replacement. The association of TFR and above or below replacement migration is not just a matter of lower M_R being associated with higher TFR: there is also a weak positive correlation between TFR and net migration rate.

Accuracy of Rule of Thumb Estimators

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[19] This is shown by Eq. (18).

Two simple, approximate models for estimating TFR_R are evaluated. Model 1 is the simple regression line of TFR_R on the net migration rate:

$$TFR_{R,A} = 2.0849 - 0.1142\, M_A \times 1000/POP_A \qquad \text{(M1)}$$

Model 1 requires only two data inputs, net migration and population, both of which are readily available from United Nations, World Bank or national official statistical agency websites, and elementary arithmetic. The predicted value for zero net migration is close to the contemporary values of (zero migration) replacement for countries with low mortality[20]; it appears consistent with the '2.1 rule'. However, unlike the '2.1 rule', Model 1 conveys clearly that the below replacement level of fertility which is consistent with sustaining the current population size depends on which level of net migration is maintained over the long run; the higher the level of net migration the lower the required TFR. Overall Model 1 produces reasonably accurate estimates of TFR_R. Across the 22 countries the MAE is 0.06 and the MAPE is 4.28%. The errors from Model 1 reflect the variability in the Index of Births I_B and life expectancy after migration eNM (Table 3). The overestimation of TFR_R in absolute terms is greatest for Netherlands, Belgium and Australia, all of which have relatively large values for I_B and eNM, due to their relatively young and predominantly female net migration, and (for I_B), relatively old ages at birth. The greatest underestimation of TFR_R is for Canada, New Zealand and Germany, which all have low values for I_B[21] and below average values of eNM.

Model 2 substitutes the mean values across all 22 countries of $\frac{TFR_A}{NRR_A}$ (2.0732), $(I_{eNM,j} \times \alpha_j)$, (0.3376 for males and 0.3491 for females), $I_{B,A}$ (0.3204), s_1 (0.5137) and s_2 (0.4863) into Eq. (17):

$$TFR_{R,A} = 2.0732 \times \frac{POP_A - M_A(0.3376\, e_{0,1,A} + 0.3491 e_{0,2,A})}{POP_A - M_A(0.3376\, e_{0,1,A} + 0.3491 e_{0,2,A}) + 0.3204 \times 2.0732 M_A\,(0.5137 e_{0,1,A} + 0.4863 e_{0,2,A})} \qquad \text{(M2)}$$

Model 2 also relies on readily available inputs (i.e. population size, total net migration and life expectancy at birth for each sex) and involves straightforward calculation. Unlike Model 1 Model 2 conveys that TFR_R depends on life expectancy, as well as net migration. It communicates the typical value of (with zero migration) exact replacement (i.e. 2.0732) slightly more accurately than Model 1. However, it involves a more complex expression and

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[20] Across all 22 countries in this study the mean for (conventional zero migration) exact replacement is 2.0732.

[21] The causes of low value of I_B differ between countries. In Canada a relatively old age profile for female net migration is the major reason, whereas for New Zealand and Germany the high sex ratio of net migration is of greater importance.

requires more time to calculate[22]. The improvement in accuracy offered by Model 2 relative to Model 1 is quite small (Table 4).

Discussion

This paper shows the heterogeneity of the long run prospects for population growth of those More Developed Countries which have both a Total Fertility Rate below 2.1 and positive net international migration and the heterogeneity of the fertility levels and net migration levels which are consistent with zero long run population growth. For most such countries it considers, continuation of current fertility, mortality and migration is conducive to long run population increase, in some cases to several times the current size. In others, continuation of the current levels would lead to a near extinction of the population. These patterns are unaffected by the initial population age structure. Accordingly, countries in which fertility is below the conventional (zero migration) replacement level can be categorised by the population growth implication of that TFR in conjunction with the coinciding net international migration and mortality patterns for the same country and time as follows:

1) Long run extinction. i.e. where the TFR is below the (zero migration) exact replacement level and net migration is negative (e.g. Belarus, Bulgaria, China, Croatia, FYR Macedonia, Greece, Poland, Portugal, Romania).
2) Long run reduction. i.e. Where the current TFR is below the *With Current Migration Replacement TFR* (TFR_R) (e.g. Finland, Hong Kong, Hungary, Italy, Japan, Netherlands, Republic of Korea, Slovakia).
3) Long run increase. i.e. where the current TFR is above TFR_R (Austria, Australia, Belgium, Canada, Denmark, France, Germany, New Zealand, Norway, Singapore, Sweden, Switzerland, UK, USA). This category encompasses a ‘fertility superfluous’ subcategory.

Long-cherished formulations of the demographic transition, which ignore the implication of migration, are therefore deficient as frameworks for understanding ongoing population growth trends in ‘post-transitional’ populations (Kirk 1996; Notestein 1945).

This study shows, for the first time, that the TFR level which, in combination with continued constant amounts of migration and constant mortality rates, has a long run zero population growth (from current size) varies widely between populations with net

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[22] A similar formulation with 0.5 substituted in place of all α_j and s_j and 0.6880 substituted in place of $I_{eNM,\,j}$ has virtually identical accuracy to the formulation in M2, and may be preferred on the basis of its greater simplicity.

immigration. For some countries with low migration relative to population, for example Japan, Korea, Hungary and Slovakia, TFR_R is close to the conventional '2.1' exact replacement. However this paper also provides examples of other countries, such as Singapore, Norway and Australia, which even were they to have 'lowest-low' levels of fertility, would experience long run population growth if (at least) the current net migration and life expectancies also are maintained over time. Such heterogeneity shows the importance of considering the implication of fertility level on a country-specific basis, as opposed to with 'one size fits all' rules, and to consider fertility level jointly in conjunction with the prevailing (or prospective future) migration and mortality levels, as opposed to in isolation (Ryder 1997). The examples of Singapore, Norway and Australia call into question whether there is any critical low level of fertility which is synonymous with long run population decline, at least for high income and small or medium-sized countries (Kohler et al. 2002; Lutz et al. 2006).

Across a range many countries with TFRs below 2.1 in recent years, raising fertility levels either has been an objective of public policy or at least has been a matter of public debate (Gauthier 2007; Jones and Hamid 2013; Lopoo and Raissian 2018; McDonald 2006a, b; Parr and Guest 2011; Smith 2019; UNPD 2013). Since for a majority of such countries positive net immigration is a more likely prospect than zero migration, the TFR_R is a more pertinent indicator of the fertility level needed to prevent population decrease than the conventional (zero migration) replacement level (UNPD 2017). This paper shows that for most of the English-speaking, Northern European and Western European countries it would be unnecessary even to maintain the current TFR, let alone to raise it to 2.1, in order to prevent a long run population decrease, assuming maintaining net immigration around the 2011-2015 level remains feasible. For Singapore too, a country with a history of pronatalist policy, this paper shows that, provided it maintains at least its current net immigration and life expectancies, raising fertility above the current level is unnecessary for the achievement of positive long run population growth (Jones and Hamid 2015). Indeed, Singapore's TFR could fall to considerably lower levels and positive population growth would still be in prospect.

Australia provides an example of a country in which recent past public policy appears to have been motivated at least in part by pronatalism. In the mid-2000's the Australian Government introduced a range of more generous family policies, including the introduction of a 'Baby Bonus', and some senior public figures argued the need for a higher birth rate (Heard 2006; Parr and Guest 2011). One was Malcolm Turnbull, who later became

Australia's Prime Minister, and who wrote in a newspaper article: "this trend of declining fertility, in the absence of a massive increase in immigration, will result in our population declining in absolute terms and over time, we will simply die out" (Turnbull 2002). Had the measures proposed in this paper been calculated at the time they would have shown the then TFR exceeded TFR_R (1.52) by 0.2 births per woman, and the then net migration was twice M_R (55.0 thousand). One can only speculate on whether such information would have facilitated better informed deliberation on the need to try to increase fertility levels (or to change immigration): irrespective of whether or not one agrees with the need for pronatalist measures, it is important that public debate is not led by misinformed comment about population growth prospects.

The lack of recognition of which below replacement TFR would be adequate for preventing population decrease, or even of there being any such a level below 2.1, continues despite recognition of the possibility of a stationary population being produced by below replacement fertility and positive net immigration dating back as far as Pollard (1973). It may be that the complexity of the formulation of stationary population models (for example using matrix algebra or integral calculus) which has been used in the literature has prevented the dissemination of the understanding of such possibilities for stabilising population size to the wider public, or even to the many students of population who are drawn from disciplinary backgrounds which typically do not require a mastery of such advanced mathematics. Demographers should stop "sitting in the corner being clever with themselves", to paraphrase a remark by former Australia Prime Minister Paul Keating (Keating cited in Atfield 1993). The cultivation of awareness of an approximate 'ball park' value for the TFR which, in combination with estimates of migration level and life expectancy, is consistent with preventing long run population decline should provide a sufficient basis for guiding public debate and policy: precision is of little consequence. The relatively simple, if approximate, rule-of-thumb, presented in this paper, may enhance the 'teachability' of such levels to a wider audience, and, by doing so, reduce the likelihood of misinformed public debate and policy.

The international evidence of the effects of policy initiatives on fertility levels is contested (Gauthier 2007; Lopoo and Raissian 2018; McDonald 2006a; 2006b). However there is no suggestion in the literature that large increases in fertility can be readily delivered by policy changes. For many More Developed Countries with TFRs below 2.1, a substantial change to net immigration may be more achievable through policy intervention than a substantial change to fertility. This paper also presents levels of net immigration, which in conjunction with the continuation of current fertility and mortality would equate the long-run future population with the current population. Current international practice appears to be to report net migration without regard to its long-run population growth implications. Indexing the current annual net migration against the number needed to produce a long-run stationary population equal in size the current, if fertility and mortality remained constant (i.e. to *Migration Replacement Level* (M_R)) and against the number needed to do so if mortality remains constant and the TFR is zero (i.e. to *Fertility Superfluous Level* (M_0)) could potentially add perspective to the population growth implication of a country's current net migration. This paper shows that for most of the countries with TFRs below 2.1, it is decrease in net migration, and not increase, which is consistent with long run maintenance of population size. Singapore exemplifies the feasibility of a country exceeding M_R even with a TFR as low as 1.23. and Germany, despite its larger population size, the feasibility of doing so with a TFR of 1.44. However, caution is needed in relation to the feasibility of other countries with similarly low fertility levels being able to do so, especially those with an even larger population size, those which are less attractive as a destination for migrants, or those in which the prevailing attitude to immigration is less accepting.

The following four properties of P_A, TFR_R and M_R in particular are especially worth noting. Firstly, their values apply to distant future time and hypothetical stability in data input values. The presentation of TSP size (and ages) may be complementary to population projections showing the more immediate population growth implications of continuation of current (or indeed of any combination of constant) fertility, migration and mortality. The 'value add' of presenting the TSP size is to illustrate a longer run implication, which is unaffected by the initial age-sex distribution of the population. In theory the evaluation of future demographic (and other) scenarios should consider all future time, and hence, both the more immediate and coherent more distant demographic prospects (Cutler et al. 1990; Parr and Guest 2014). The weights of importance to attach to more immediate and more distant prospects are a matter of philosophical debate and assessments vary from-country-to-country, and, in practice, whether due to convenience, avoidance of uncertainty or egocentricity, the distant future may be given little weight (Samuelson 1958; Wang et al. 2016).

Secondly, the values of P_A, TFR_R and M_R may be volatile for some countries. For some countries, migration levels have fluctuated considerably over time. A notable example is Spain, which over 2005-2010 had the third largest net immigration worldwide and over 2010-2015 had negative net migration (UNPD 2017). Such fluctuation over time affects the stability of TFR_R. Life expectancy at birth has generally increased over time (Parr et al. 2016;

UNPD 2017). Future mortality improvement would lower the TFR_R and M_R corresponding to specified levels of migration and fertility respectively. To the extent that further mortality improvement is expected in the future, values of TFR_R and M_R corresponding to current mortality levels, may be viewed as upper limits to the range of values for the respective TFR and net migration numbers under which long run future population size would exceed the current level (De Beer et al. 2017; Dong et al. 2017; Lenart and Vaupel 2017). Regarding fertility, even small changes to TFR levels which lie below (zero migration) exact replacement level can lead to substantial changes in M_R. The length of the time period over which data input values are averaged will also affect the volatility of TFR_R and M_R, and the plausibility of such values as indicators of future prospects.

Thirdly, the values of some of the data the inputs used in calculating TFR_R and M_R may, to some extent, be jointly determined. Thus, for example, the decreases in TFR have generally been associated with increases in mean age at birth (Rindfuss et al. 2016). However, the calculation of $\mathrm{TFR_R}$ does not make any allowance for differences in ages at birth which might result from differing TFR levels. Finally, whilst under both constant TFR_R or constant M_R the TSP size will boomerang over time towards the current population size, the age profiles of both the corresponding TSPs will differ from each other and from the current population age structure (Ryder 1997; Schmertmann 1992). None of the above concerns should affect preference for use of the *With Migration Replacement TFR* ahead of ‘2.1’ as a yardstick for the population growth implication of fertility in countries with net immigration.

Appendix A: Derivation of Equation (7)

For convenience Eq. (6) is reorganised to collect terms which are scalar multiples of the TFR on one side of the equation. That is:

$$P_A = P_{1,A} + \frac{P_{2,A}}{(1-NRR_A)}$$

Hence:

$$(P_A - P_{1,A})(1\text{-}NRR_A) = P_{2,A} \qquad (A.0)$$

Hence:

$$(P_A - \mathrm{P}_{1,\mathrm{A}}) = P_{2,A} + NRR_A\,(P_A - P_{1,A}) \qquad (A.1)$$

When $P_A = POP_A$ Eq. (6.1) has the form:

$$(POP_A - P_{1,A}) = P_{2,A,R} + NRR_{R,A}\,(POP_A - P_{1,A}) \qquad (A.2)$$

Where $\mathrm{P}_{2,\mathrm{A},\mathrm{R}}$ denotes the 2nd generation component of the stationary population of size equal to corresponding to POP_A, M_A, $m_{x,j,A}$, $e_{x,j,A}$, $s_{j,A}$, $f_{x+t,A}$ and ${}_tp_{x,A}$, and $NRR_{R,A}$ and the (zero migration) net reproduction rate corresponding to $s_{1,A}$, $f_{x+t,A}$ and ${}_tp_{0,A}$ and the With Migration Replacement TFR ($TFR_{R,A}$) .

Rewriting $NRR_{R,A}$ as:

$$NRR_{R,A} = \frac{TFR_{R,A}NRR_{r,A}}{TFR_{R,A}} \qquad (A.3)$$

And rewriting $P_{2,A,R}$ as:

$$P_{2,A,R} = \frac{TFR_{R,A}P_{2,R,A}}{TFR_{R,A}} \qquad (A.4)$$

Since specific values of M_A, $m_{x,j,A}$, $e_{x,j,A}$, $s_{j,A}$, $f_{x+t,A}$ and ${}_tp_{x,A}$ are common to the calculations of $NRR_{R,A}$, NRR_A, $TFR_{R,A}$, TFR_A, $P_{R,2,A}$ and $P_{2,A}$

$$\frac{NRR_{R,A}}{TFR_{R,A}} = \frac{NRR_A}{TFR_A} \qquad (A.5)$$

and

$$\frac{P_{R,2,A}}{TFR_{R,A}} = \frac{P_{2,A}}{TFR_A} \qquad (A.6)$$

Substituting from Eq. (A.3), (A.4), (A.5) and (A.6) into Eq. (A.2)

$$POP_A - P_{1,A} = \frac{TFR_{R,A}\,P_{2,A} + (POP_A - P_{1,A})TFR_{R,A}NRR_A}{TFR_A} = \frac{TFR_{R,A}\,[P_{2,A} + (POP_A - P_{1,A})NRR_A]}{TFR_A} \qquad (A.7)$$

Multiplying both sides of Eq. (A.7) by TFR_A gives:

$$TFR_A\,(POP_A - P_{1,A}) = TFR_{R,A}\,(P_{2,A} + ((POP_A - P_{1,A})\,NRR_A)) \qquad (A.8)$$

and hence Eq. (7):

$$TFR_{R,A} = \frac{TFR_A(POP_A - P_{1,A})}{\left(NRR_A(POP_A - P_{1,A})\right) + P_{2,A}} = \frac{TFR_A}{NRR_A} \times \frac{POP_A - P_{1,A}}{POP_A - P_{1,A} + \frac{P_{2,A}}{NRR_A}}$$

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Fig. 1: Comparison of With Migration Replacement Total Fertility Rate to Total Fertility Rate to 22 Countries 2011-15

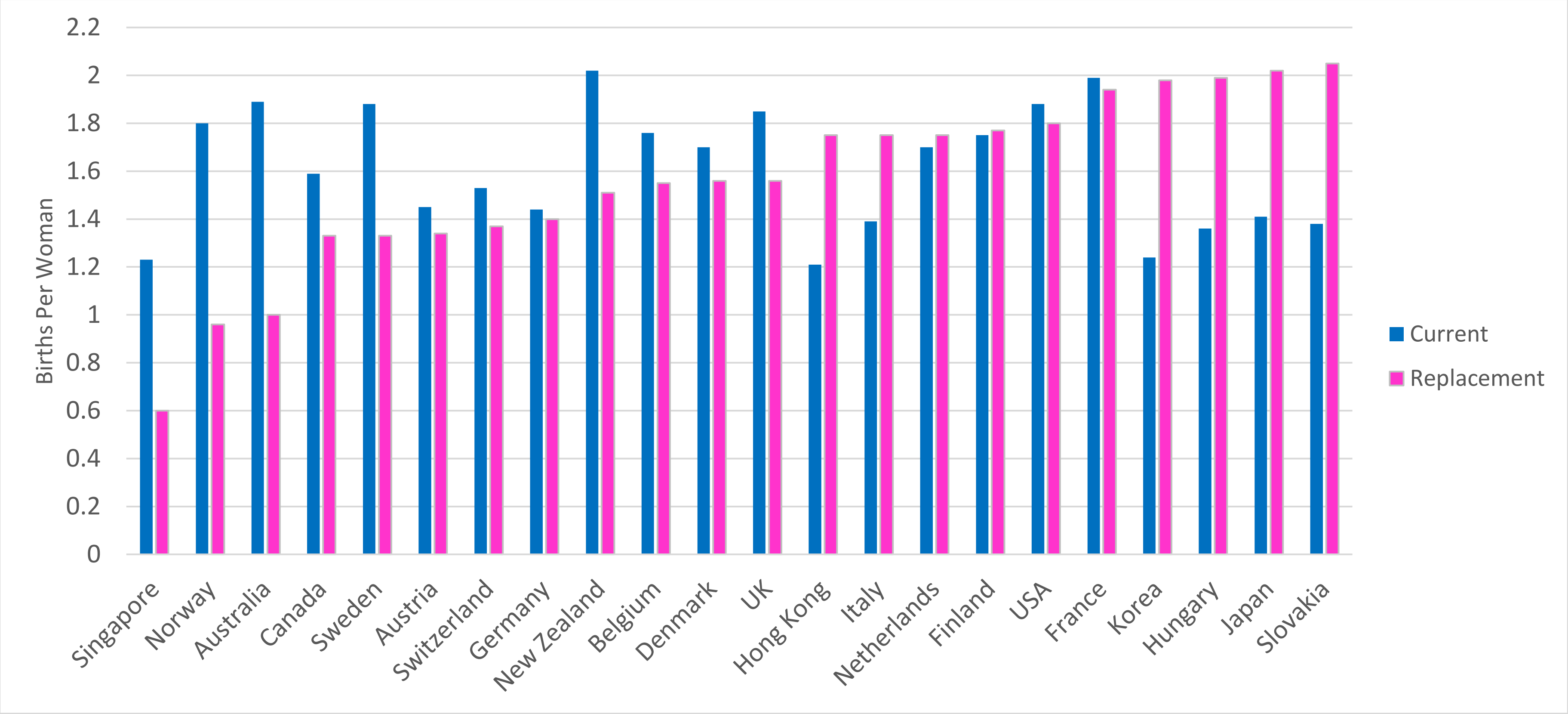

Fig. 2: With Migration Replacement Total Fertility Rate Plotted Against Net Migration Rate for 2011-2015

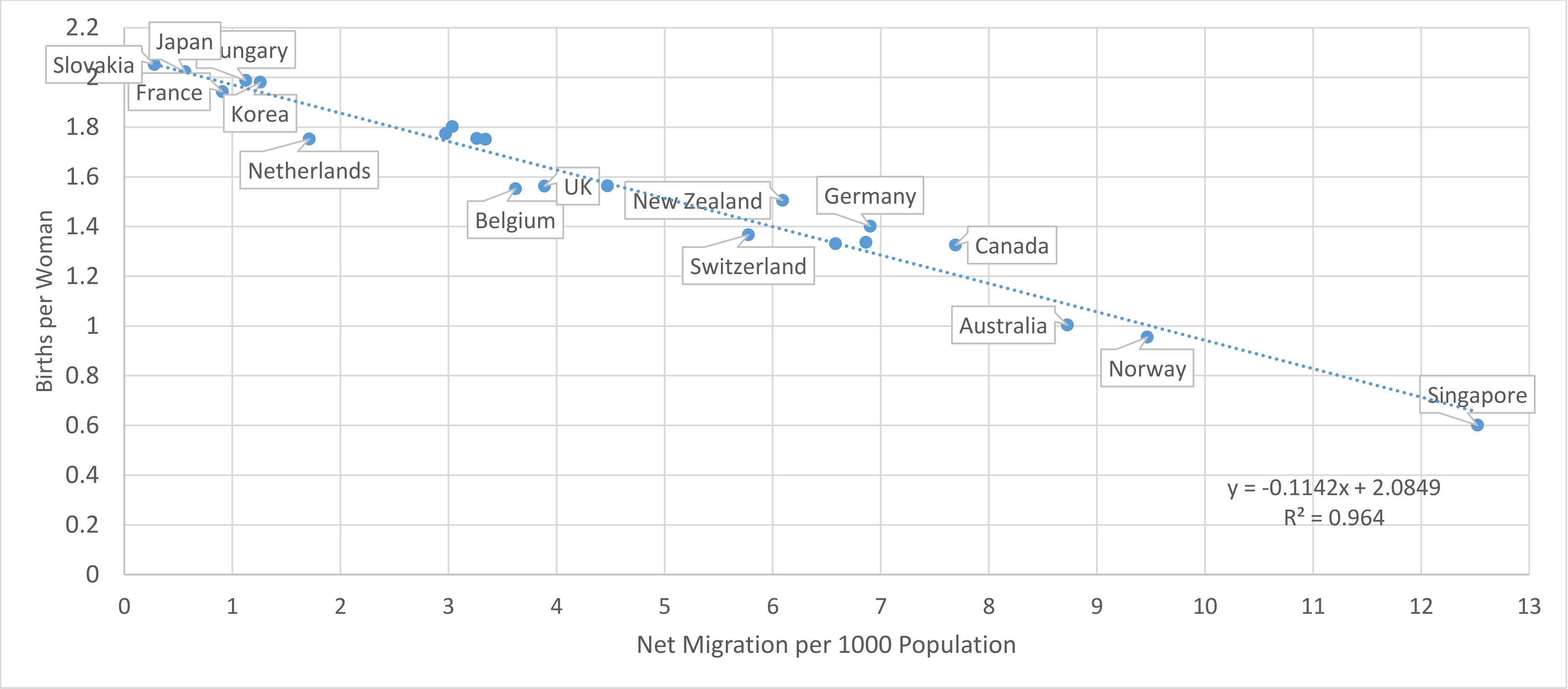

Fig. 3: Replacement Migration per 1000 Population Plotted Against Total Fertility Rate for 2011-2015

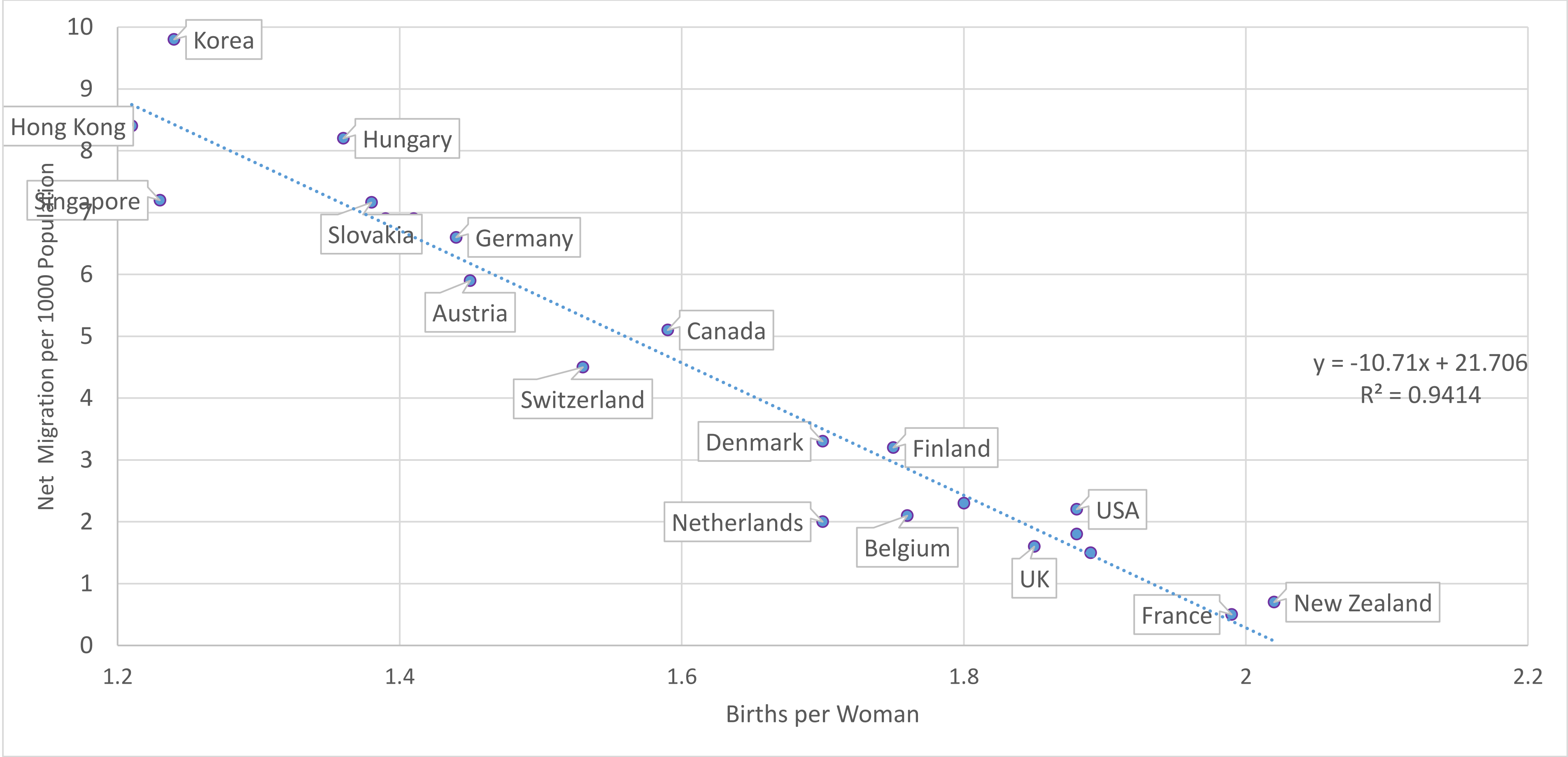

Table 1: Summary Measures of Input Demographic Data: Selected Countries 2011-2015

Country	Total Population (Millions)	TFR (per woman)	NRR (per woman)	Mean Age at Birth (years)	Net Migration (000s)	Net Migration Rate (per 1000)	Sex Ratio of Net Migration (males per 100 females)	Mean Age at Net Migration (years)		Life Expectancy at Birth (years)	
								Male	Female	Male	Female
Australia	23.1	1.89	0.91	30.6	201.8	8.7	85.9	23.2	25.8	80.3	84.4
Austria	8.5	1.45	0.70	30.3	58.2	6.9	117.6	24.2	25.0	78.6	83.8
Belgium	11.2	1.76	0.85	30.1	40.4	3.6	81.6	14.8	19.5	78.1	83.2
Canada	35.2	1.59	0.77	30.4	270.4	7.7	94.2	29.9	30.4	79.7	83.9
Denmark	5.6	1.70	0.82	30.8	25.1	4.5	116.2	25.0	24.3	78.3	82.4
Finland	5.4	1.75	0.85	30.4	16.2	3.0	113.8	26.1	27.4	78.0	84.1
France	65.8	1.99	0.97	30.2	59.7	0.9	41.6	23.5	22.0	79.0	85.6
Germany	80.7	1.44	0.69	30.4	556.7	6.9	148.1	25.6	25.3	78.1	83.0
Hong Kong	7.2	1.21	0.58	31.8	23.4	3.3	51.9	29.9	33.7	81.1	86.7
Hungary	9.9	1.36	0.65	29.5	11.1	1.1	137.3	28.9	30.7	72.2	79.1
Italy	60.2	1.39	0.67	31.5	201.3	3.3	90.0	27.4	32.6	80.3	85.2
Japan	127.4	1.41	0.68	30.8	71.6	0.6	111.6	30.3	32.1	80.2	86.6
Korea	51.1	1.24	0.60	31.5	64.2	1.3	157.5	29.7	34.4	78.1	84.6
Netherlands	16.8	1.70	0.82	31.0	28.8	1.7	47.4	-0.51	21.3	79.5	83.2
New Zealand	4.4	2.02	0.97	29.9	27.1	6.1	134.6	24.2	25.0	79.5	83.2
Norway	5.1	1.80	0.87	30.5	48.1	9.5	86.7	25.0	25.3	79.8	83.8
Singapore	5.4	1.23	0.59	31.3	67.6	12.5	98.8	24.9	26.4	80.1	84.5
Slovakia	5.4	1.38	0.67	28.8	1.5	0.3	167.6	31.6	18.7	72.9	80.1
Sweden	9.6	1.88	0.91	30.9	63.2	6.6	111.0	24.3	24.7	80.1	83.8
Switzerland	8.1	1.53	0.74	31.6	46.7	5.8	85.2	23.7	25.7	80.7	85.0
UK	64.1	1.85	0.90	30.0	249.3	3.9	85.7	24.3	24.8	79.2	82.9
USA	316.2	1.88	0.90	28.7	959.8	3.0	94.1	29.5	32.0	76.4	81.2
Mean	42.1	1.61	0.78	30.5	140.6	4.6	102.6	24.9	26.0	78.6	83.7

Table 2: Terminal Stationary Population Size, Migration Replacement TFR, Replacement Net Migration and Fertility Superfluous Net Migration: Selected Countries 2011-2015

Country	Terminal Stationary Population (*TSP*)		Migration Replacement TFR (TFR_R)		Annual Replacement Migration (M_R)		Fertility Superfluous Level (M_0)	
	TSP Size (Millions)	Ratio of TSP Size to Current Population	TFR_R (per woman)	Ratio of Current TFR to TFR_R	M_R (000s)	M_R per 000 Current Population	M_0 per 1000 Current Population	Ratio of Current Net Migration to M_0
Australia	134.5	5.82	1.00	1.88	34.7	1.5	17.1	0.51
Austria	10.0	1.17	1.34	1.09	49.6	5.9	17.5	0.39
Belgium	19.1	1.71	1.55	1.14	23.7	2.1	15.7	0.23
Canada	53.4	1.52	1.33	1.20	178.0	5.1	18.9	0.41
Denmark	7.7	1.37	1.56	1.09	18.3	3.3	17.8	0.25
Finland	5.0	0.93	1.77	0.99	17.5	3.2	18.2	0.16
France	112.4	1.71	1.94	1.03	34.7	0.5	16.2	0.06
Germany	85.0	1.05	1.40	1.03	528.3	6.6	18.1	0.38
Hong Kong	2.8	0.39	1.75	0.69	60.4	8.4	18.8	0.17
Hungary	1.4	0.14	1.99	0.68	81.3	8.2	21.1	0.05
Italy	29.4	0.49	1.75	0.79	413.4	6.9	18.7	0.18
Japan	10.4	0.08	2.02	0.70	880.2	6.9	18.9	0.03
Korea	6.5	0.13	1.98	0.63	500.5	9.8	20.0	0.06
Netherlands	14.3	0.85	1.75	0.97	33.7	2.0	14.7	0.12
New Zealand	40.0	9.00	1.51	1.34	3.0	0.7	18.7	0.33
Norway	20.7	4.07	0.96	1.88	11.8	2.3	17.6	0.54
Singapore	9.4	1.73	0.60	2.05	39.0	7.2	17.5	0.72
Slovakia	0.2	0.04	2.05	0.67	38.8	7.2	19.9	0.01
Sweden	35.9	3.74	1.33	1.41	16.9	1.8	17.2	0.38
Switzerland	10.5	1.29	1.37	1.12	36.1	4.5	17.0	0.34
UK	153.3	2.39	1.56	1.19	104.4	1.6	17.5	0.22
USA	430.9	1.36	1.80	1.04	704.4	2.2	20.0	0.15
Mean	54.2	1.86	1.56	1.00	173.1	4.4	18.1	0.26

Table 3: Selected Values of Metrics Related to Terminal Stationary Population Size: Selected Countries 2011-2015

Country	TSP Births Index (I_B)	TFR after Net Migration (*TFRNM*)	Net Migration TFR Index (I_{TFRNM})	Life Expectancy after Net Migration (*eNM*)			Index of Life Expectancy after Net Migration (I_{ENM})		
				Male	Female	All Migrants	Male	Female	All Migrants
Australia	0.36	1.29	0.68	57.8	59.3	58.6	0.72	0.70	0.71
Austria	0.30	0.98	0.68	55.1	59.2	57.0	0.70	0.71	0.70
Belgium	0.43	1.41	0.80	63.4	64.0	63.7	0.81	0.77	0.79
Canada	0.26	0.84	0.53	51.1	54.4	52.8	0.64	0.65	0.65
Denmark	0.33	1.25	0.73	54.1	58.6	56.2	0.69	0.71	0.70
Finland	0.28	1.07	0.61	52.8	57.3	54.9	0.68	0.68	0.68
France	0.40	1.14	0.57	56.0	64.2	61.4	0.70	0.75	0.74
Germany	0.26	0.96	0.66	53.3	58.2	55.3	0.68	0.70	0.69
Hong Kong	0.28	0.54	0.44	52.1	53.6	53.1	0.64	0.62	0.63
Hungary	0.26	0.83	0.61	45.2	50.2	47.3	0.63	0.63	0.63
Italy	0.27	0.73	0.53	53.7	53.3	53.5	0.67	0.62	0.65
Japan	0.26	0.79	0.56	50.9	55.3	53.0	0.64	0.64	0.64
Korea	0.21	0.69	0.56	49.4	51.1	50.1	0.63	0.60	0.62
Netherlands	0.57	1.44	0.85	79.6	62.4	67.9	1.00[+]	0.75	0.83
New Zealand	0.26	1.25	0.62	52.5	54.7	53.4	0.66	0.66	0.66
Norway	0.34	1.18	0.65	54.8	58.9	57.0	0.69	0.70	0.70
Singapore	0.34	0.85	0.69	55.8	58.5	57.2	0.70	0.69	0.69
Slovakia	0.29	1.08	0.78	43.1	62.5	50.4	0.59	0.78	0.67
Sweden	0.32	1.29	0.68	56.6	59.6	58.0	0.71	0.71	0.71
Switzerland	0.36	1.04	0.68	57.5	59.8	58.7	0.71	0.70	0.71
UK	0.40	1.40	0.76	55.7	58.7	57.3	0.70	0.71	0.71
USA	0.27	1.00	0.53	49.1	50.9	50.0	0.64	0.63	0.63
Mean	0.32	1.05	0.65	54.5	57.5	55.8	0.69	0.69	0.69

+ For Netherlands cumulative net migration is negative for all ages above 40. For explanation of values exceeding 1 see footnote 12.

Table 4: Rule of Thumb Estimate and Error of Migration Replacement Total Fertility Rate from Model 1 and Model 2: Selected Countries 2011-2015

Country	Model 1		Model 2	
	Estimate of TFR_R	Error	Estimate of TFR_R	Error
Australia	1.09	0.09	1.07	0.07
Austria	1.30	-0.04	1.30	-0.04
Belgium	1.67	0.12	1.67	0.12
Canada	1.21	-0.12	1.20	-0.13
Denmark	1.57	0.01	1.57	0.01
Finland	1.74	-0.03	1.74	-0.03
France	1.98	0.04	1.97	0.03
Germany	1.30	-0.10	1.30	-0.10
Hong Kong	1.71	-0.04	1.70	-0.05
Hungary	1.96	-0.03	1.96	-0.03
Italy	1.71	-0.04	1.69	-0.06
Japan	2.02	0.00	2.01	-0.01
Korea	1.94	-0.04	1.93	-0.05
Netherlands	1.89	0.14	1.88	0.13
New Zealand	1.39	-0.12	1.38	-0.13
Norway	1.00	0.04	0.99	0.03
Singapore	0.66	0.06	0.62	0.02
Slovakia	2.05	0.00	2.04	-0.01
Sweden	1.33	0.00	1.32	-0.01
Switzerland	1.42	0.05	1.41	0.04
UK	1.64	0.08	1.64	0.08
United States	1.74	-0.06	1.74	-0.06
MAE		0.06		0.06
MAPE (%)		4.20		3.88